The Staple Inn Actuarial Society

Frictional Costs

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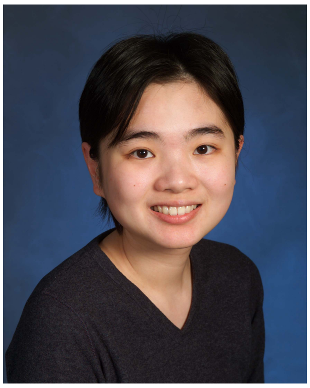

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1 INTRODUCTION

1.1 Definition of Frictional Costs

In financial literature the term *frictional costs* is often used for transaction costs. In this paper we define frictional costs as follows:

Frictional costs are positive, irrecoverable cashflows away from shareholders and bondholders that have a convex (U-Shaped) relationship with profit.

1.2 Purpose of the Paper

Deflators and other risk-neutral techniques have been successful in valuing financial instruments. The next logical step is to use them to value companies using internal cashflow models. However, modellers have found that valuations using internal cashflow models have tended to overstate valuations observed in the market.

Some frictional costs are already included in cashflow models. The remaining frictional costs are seldom included. We believe the principal reason for the lack of reconciliation to market values is that cashflow models being used do not capture the remaining frictional costs. We also believe that model approximations and management optimism can overstate the cashflows to shareholders.

Even for those that already accept this argument there has been the problem of how to include the residual frictional costs in the cashflow model. In this paper we introduce a technique that will help modellers make their cashflow models more realistic.

1.3 Definition of Frictional Cost Functions

The technique we will introduce is the *frictional cost function*. We define it as follows.

A frictional cost function is a positive and convex function of profit (or other suitable variable) with one free parameter that can vary the severity of the frictional cost function.

The name *frictional cost function* is slightly misleading because we use it to capture the model approximations and management optimism, in addition to the residual frictional costs.

1.4 Traditional Approach

Many attempts at finding ways to reconcile cashflow models to market consistent company valuations use extra margins on a deterministic risk discount rate. Justifications for these margins are often based on a reward for bearing unsystematic risk. They can even be fudge factors just to make the model reconcile to market values.

However, margins for unsystematic risk are inconsistent with financial economics. Our work suggests that the motivation to find a reward for unsystematic risk can be understood by using financial economics with frictional cost functions.

1.5 Frictional Cost Approach

We define the *idealistic profit*, P, as the profit figure output from an existing stochastic cashflow model. Our method for making the cashflow model realistic is to deduct a single cashflow from the idealistic profit to obtain the *realistic profit*. The cashflow deduction is calculated using a frictional cost function, $\theta(P)$.

The risk-adjusted price, V_P, is then equal to the expectation of the product of the *realistic* profit and state price deflator, as shown in the following equation.

$$V_P = E[D(P - \theta(P))]$$

Equation 1-1

The result is a risk-adjusted valuation method that allows for residual frictional costs, model approximations and management optimism.

1.6 Benefits of the New Approach

The new approach has several virtues.

- Consistency with financial economics
- Easy integration with deflator models
- Better quantification of the value of risk mitigation
- Better quantification of the value of diversification
- Better quantification of a business unit's contribution to company profits

1.7 Structure of the Paper

Our challenge in writing this paper was to account for the different levels of knowledge that people have on this topic. We have tried to find a balance between writing a paper that beginners are able to follow and that more advanced readers will not find tedious.

Our solution is to keep the main body of the paper to a few select examples and refer the reader to appendices as required. Where possible, we have attempted to use the simple, but popular, Feast & Famine example familiar to readers of the 2001 SIAS paper Modern Valuation Techniques by Jarvis, Southall & Varnell. There are occasions when a three state model is required so we have introduced the Feast, Fine and Famine model.

To help the readers who wish to follow the calculations, we have shown the working for the calculations in the first cell of some tables. For readers who want to follow the calculation is more detail, a spreadsheet with all the calculations set out is available on request from the authors.

In Section 2 we summarise the financial economics that is assumed in the rest of the paper. We summarise the main assumptions and refer the reader to the relevant appendix or other papers for more detail.

In Section 3 we use the Modigliani & Miller result to motivate the focus on frictional costs. We also suggest a suitable form for the frictional cost function.

In Section 4 we introduce the concept of risk measures as applied to insurance and financial pricing. We give examples of risk measures and discuss the causes of differences between the insurance and finance pricing axioms.

In Section 5 we introduce a simple risk measure for frictional costs. We justify a suitable set of risk measure axioms and show how the risk measure we have chosen satisfies each one. Finally we show a more realistic frictional cost function that is used in practice.

In Section 6 we use a simple three state model to demonstrate some of the features of frictional cost functions. We look at how the frictional cost function approach to allocating frictional costs captures the risk contribution of particular business unit. We also see how to reverse-engineer equivalent discount rate adjustments.

In Section 7 we use the three state model to explore how unsystematic risk impacts valuation through frictional cost functions by altering the expected value of frictional costs and introducing an element of systematic risk. We finish by considering how economic & operational models can be combined.

In Section 8 we review some common management techniques and discuss how frictional cost functions can help explain some of the anomalies between them and finance theory.

In Section 9 Rolf van den Heever considers the implications of frictional cost functions for General Insurance. He considers strategic decisions, valuation and non-systematic risk.

In Section 10 Olivier Allen considers the implications of frictional cost functions for the Petroleum Industry. He considers the implications for valuing a portfolio of North Sea oil fields.

Appendix A has an introduction to utility theory.

Appendix B has a description of securitisation.

Appendix C has an introduction to systematic risk and shows how it can be measured in a deflator model.

Appendix D has an example of the Modigliani & Miller result.

Appendix E has a description of the mathematical properties required for a valid frictional cost function.

1.8 Originality

This paper does not contain new material. It has drawn extensively on published financial economics and actuarial papers. Our aim has been to draw together this material and present it in a way that the reader will find easier to follow.

1.9 Acknowledgements

We would like to thank three people who have made a major contribution to helping us produce this paper. Firstly to Andrew Smith, for his support, guidance and, not least, for developing the mathematical techniques at the core of this paper. Secondly to Frances Southall for the many hours she spent reviewing our paper and for many useful discussions. Finally to Stavros Christofides, for the introduction to frictional costs and for many useful discussions on this and other topics.

Special thanks also to our two contributors who have taken the time to understand the techniques that are discussed in this paper, and have started to apply them to their own areas of work. Firstly to Olivier Allen, for many useful discussions and ideas on applications to the Petroleum Industry. Secondly to Rolf van den Heever, for many useful discussions and his ideas on what these techniques mean for General Insurance. Each one has contributed a section on applications in their area of work.

Thanks also to our other reviewers; Phil Joubert, Tony Jeffery, Stephen Carlin, Victor Mirkin, Harold Clarke, and David Holland.

2 FINANCIAL ECONOMICS

2.1 Introduction

In this section we summarise some topics in financial economics that are relevant to the paper. The purpose is to ensure that we do not make any unreasonable assumptions about the knowledge of the reader.

We have provided several supporting appendices to help those unfamiliar with these topics. We hope this will help readers find a roadmap through the paper that suits their background.

2.2 Terminology

In this paper we make references to returns and volatilities so it is useful to clarify some definitions. All returns are total returns and use a single period compounding convention. We can express this as follows, where $Value_{End}$ is assumed to include investment income.

$$Return = \frac{Value_{End}}{Value_{Start}} - 1$$

Equation 2-1

- *Return* means *realised return measured after the event.*
- *Excess Return* means *return less the risk-free return*.
- *Expected Return* means *expectation of the return distribution.*
- *Return Volatility* means *standard deviation of the return distribution.*
- *Risk Premium* means *expected return less the risk-free return*.

2.3 Key Concepts from Financial Economics

2.3.1 Arbitrage-Free Pricing

Arbitrage-free pricing means two assets that pay the same amount, at the same time and under the same circumstances should be valued at the same price.

This is covered in Chapter 2 of Modern Valuation Techniques.

2.3.2 State Price Deflators

State Price Deflators are an arbitrage-free valuation technique (equivalent to risk-neutral methods) using real-world probabilities. Prices calculated using deflators are called *risk-adjusted prices*. The deflator valuation equation is shown below where X is a risky cashflow, D is the state price deflator and V_X is the risk-adjusted price.

$$V_X = E[DX]$$

Equation 2-2

This is covered in Chapter 4 of Modern Valuation Techniques.

2.3.3 Utility Functions

Typical investors have the following traits:

- Investors prefer more wealth to less
- Investors are risk averse

Utility functions are used to quantify traits of investor behaviour. Using utility functions it is possible to show that typical investors require a positive risk premium as compensation for an uncertain outcome.

This is illustrated in Appendix A.

2.3.4 Systematic & Unsystematic Risk

Readers may know systematic risk as undiversifiable risk and unsystematic risk as diversifiable risk.

Investors require a risk premium for systematic risk but do not require one for unsystematic risk. This is because they can avoid unsystematic risk through diversification.

Readers may be familiar with the systematic risk definition from the Capital Asset Pricing Model (CAPM). In CAPM the systematic risk is that part of an asset's risk correlated with the market portfolio.

In a deflator model systematic risk is that part of an asset's risk that is correlated with the deflator. Unsystematic risk is the residual risk that is uncorrelated with the deflator. The definition using the deflator is more general than the CAPM definition. This is because it captures the covariance of the asset with the marginal utility of the optimal portfolio of every risk-averse investor.

In a deflator model the systematic risk for an asset is the negative of the covariance of the asset return, R_X, with the deflator, D, as shown below.

$$\textit{Systematic Risk} = -COV[D, R_X]$$

Equation 2-3

The risk premium on an asset is then given by the following expression, where r is the risk-free rate.

$$E[R_X] - r = -\frac{COV[D, R_X]}{E[D]} = -(1+r)COV[D, R_X]$$

Equation 2-4

The proof of this is shown in Appendix C of this paper.

Deflator or risk-neutral valuation methods only adjust prices for systematic risk. Risky cashflows uncorrelated with the deflator constitute unsystematic risk and are discounted at the risk-free rate, r, as shown below.

$$P = E[DX] = COV[DX] + E[D]E[X] = E[D]E[X] = \frac{1}{1+r}E[X]$$

Equation 2-5

2.3.5 Transfer of Wealth Between Bondholder & Shareholders

All other things equal an increase in the profit volatility results in a transfer of wealth from the bondholders to the shareholders. The transfer of wealth can be understood qualitatively as an increase in the value of the limited liability option of the shareholders. This increase in value is at the expense of higher default risk for the bondholders, which lowers the value of their bonds.

This can be demonstrated numerically by considering shareholder value as a call option on the overall value of the company. It is covered in Merton (1974). It can also be demonstrated using the Feast & Famine example, although we have not done so in this paper.

2.3.6 Modigliani & Miller

We summarise the Modigliani & Miller result later in the paper. However, readers who are unfamiliar with result may like to work through a numerical illustration using the Feast & Famine model in Appendix D.

2.4 Models Used in This Paper

2.4.1 Feast & Famine

Throughout this paper and the appendices we will reuse a simple two state, one time period model that will be familiar to readers of Modern Valuation Techniques. To avoid repeatedly reintroducing the model we summarise it below. When the reader sees a reference to the Feast & Famine model the figures used in calculations will be taken from here.

Asset M is a combination of Asset A & Asset B. In some places Assets A & B refer to shares in a capital market. Where this happens Asset M refers to the market portfolio. In other places Asset A & Asset B refer to Division A and Division B of Company M.

The state probabilities are as follows.

	Feast	***Famine***
Probability	0.5	0.5

Table 2-1

The deflators and asset specific cashflows are as follows.

X	***Feast***	***Famine***	***E[X]***
Deflator (D)	0.7	1.2	0.95
Asset A (A)	3	1	2
Asset B (B)	2	0.5	1.25
Asset M (M)	5	1.5	3.25

Table 2-2

The prices for each asset are calculated as follows.

X	***Feast***	***Famine***	***$V_X = E[X]$***
D·A	0.7×3 = 2.1	1.2	0.5(2.1+1.2) = 1.65
D·B	1.4	0.6	1
D·M	3.5	1.8	2.65

Table 2-3

The returns for each asset in each state are as follows.

R(X)	***Feast***	***Famine***	***E[R(X)]***
A	3/1.65-1 = 0.818	-0.394	0.5(0.818×-0.394) = 0.212
B	1	-0.5	0.25
M	0.887	-0.434	0.226

Table 2-4

The price of the risk-free asset is the expectation of the deflator, so the risk-free rate is as follows.

	Return
Risk-Free Asset	1/0.95-1 = 0.0526

Table 2-5

The risk premiums and return volatilities are as follows.

	Asset A	***Asset B***	***Asset M***
Risk Premium	0.212-0.0526 = 0.159	0.197	0.174
Return Volatility	σ(0.818,-0.394) = 0.857	1.06	0.934

Table 2-6

2.4.2 Feast, Fine & Famine

Some sections of the paper require a three state model to illustrate the concept being presented. We therefore introduce the Feast, Fine and Famine model. This is similar to the Feast & Famine model except that it includes an extra state. To save repeatedly reintroducing the model we summarise it below. When the reader sees a reference to the Feast, Fine & Famine model some figures used in calculations will be taken from here.

Asset M is a combination of Asset A & Asset B. In some places Asset A & B refer to shares in a capital market. Where this happens Asset M refers to the market portfolio. In other places Asset A & Asset B refer to Division A and Division B of Company M.

The state probabilities are as follows.

	Feast	*Fine*	*Famine*
Probability	0.333	0.333	0.333

Table 2-7

The deflators and asset specific cashflows are as follows.

X	***Feast***	***Fine***	***Famine***	***E[X]***
D	0.6	0.9	1.35	0.95
A	1.5	1.5	3	2
B	3	2	1	2
M	4.5	3.5	4	4

Table 2-8

The prices for each asset are calculated as follows.

X	***Feast***	***Fine***	***Famine***	$V_X = E[X]$
D·A	0.6×1.5 = 0.9	1.35	4.05	0.333(0.9+1.35+4.05) = 2.10
D·B	1.8	1.8	1.35	1.65
D·M	2.7	3.15	5.4	3.75

Table 2-9

The returns for each asset and state are as follows.

R(X)	***Feast***	***Fine***	***Famine***	***E[R(X)]***
A	1.5/2.10-1 = -0.286	-0.286	0.429	0.333(-0.286+-0.286+0.429) = -0.0476
B	0.818	0.212	-0.394	0.212
M	0.200	-0.0667	0.0667	0.0667

Table 2-10

The price of the risk-free asset is the expectation of the deflator, so the risk-free rate is calculated as follows.

	Return
Risk-Free Asset	1/0.95-1 = 0.0526

Table 2-11

The risk premiums and return volatilities are as follows.

	Asset A	***Asset B***	***Asset M***
Risk Premium	-0.0476-0.0526 = -0.100	0.159	0.0140
Return Volatility	σ(-0.286, -0.286,0.429) = 0.412	0.606	0.133

Table 2-12

3 FRICTIONAL COSTS

3.1 Introduction

In this section we summarise the research of Modigliani & Miller and subsequent work to motivate our opinion that frictional costs are a significant component of company valuation.

3.2 Modigliani & Miller

In 1958 Modigliani & Miller published research suggesting the following conclusions:

- Leverage (the split between debt financing and equity financing) is irrelevant to the company value.
- Dividend policy is irrelevant to the company value.

Subsequent generalisation of their work suggested the following conclusion.

- Profit volatility is irrelevant to the valuation of the company.

The conclusions were controversial when first published and have remained so to the present day. The source of the controversy is that some of its conclusions are not reflected by the experience of real companies. Most companies believe some mixture of debt and equity to be optimal. Furthermore, many companies use risk mitigation to reduce profit volatility. Subsequent research has tried to explain these discrepancies. The logical argument of the conclusions from the assumptions is considered sound. Therefore the focus of research has been on the validity of the assumptions.

One of the assumptions relates to the market.

- Financial markets are arbitrage free.

Other assumptions relate to the cashflows away from the shareholders and bondholders.

- Taxes are neutral
- No transaction costs
- No costs of financial distress
- No agency costs

There is also an assumption about new information signals.

- No new information about the future profit is provided through changes in the financing decision or dividend policy.

In the next sub-section we consider the significance of each assumption and comment on how easy it is to model.

3.3 Modigliani & Miller Assumptions

3.3.1 Financial markets are arbitrage-free.

Although short term arbitrage opportunities exist in financial markets, they tend to be exploited by investors who can react very quickly and with large amounts of capital. For other investors markets appear to be arbitrage-free.

Although arbitrage cannot always be ruled out, it is unlikely to have a significant impact on valuation. It is also difficult to build a reliable model that will indicate where arbitrage opportunities exist.

3.3.2 Taxes are neutral

Taxes are payments to the government from shareholders and bondholders.

They are neutral if they do not have a distorting effect on valuation. Tax regimes are not completely neutral because tax rates tend to rise as profits rise, but are only partially recoverable when profits fall.

Furthermore, a company might plan its strategy for a particular tax regime only to find it is tax inefficient after a change in policy or change of government.

Governments attempt to make the tax system neutral which can make this a second order effect. Sophisticated tax models are commonplace so capturing these effects explicitly is feasible.

3.3.3 No transaction costs

Transaction costs are payments made in conducting normal business. They are typically payments from shareholders and bondholders to any of the following list of beneficiaries.

- Trading Partners (Goods, Services)
- Employees (Salaries)
- Professional Partners (Consultancy, Legal Advice)

For example, insurers have significant costs due to moral hazard when they write insurance. Therefore they need to staff a claims department. The nature of employee contracts means that salaries are irrecoverable. The salary bill will often rise if the business does well as more employees are recruited. It is more difficult to reduce the salary bill if business does not do well. Similar arguments can be made for payments to other stakeholders.

The nature of transaction costs are to increase when profits are high but to have limited scope to fall when profits are low. Although convexity will undoubtedly arise from transaction costs, normal management activity tries to limit this. Therefore transaction costs are unlikely to be the most significant factor influencing valuation.

Management information systems are commonly used to model transaction costs and normal management activity is spent concentrating on how to optimise them. Consequently modelling of transaction costs is relatively well established.

3.3.4 No costs of financial distress

Costs of financial distress are payments made when the company is experiencing very low profitability. They are typically payments from shareholders and bondholders to any of the following list of beneficiaries.

- Competitors (Lost Business)
- Employees (Redundancy Costs, Pension Fund Transfers)
- Professional Partners (Administrators, Consultancy, Legal Advice)
- Investment Banks (Capital Raising)

These costs are substantial and cannot be ignored. Employees may receive redundancy packages and pension fund contributions may be required.

The most significant of these costs can be the indirect costs of customers lost to competitors as the financial position of the company worsens. This is especially true for financial companies such as insurers, which rely on a good credit rating and public confidence for their business to prosper. A financially distressed company may also have to sell its products at a discount to compensate for the additional risk of default.

In more severe cases of financial distress bankruptcy occurs. This leads to administrator or receiver costs and assets may have to be liquidated at unattractive rates. Furthermore shareholders may lose future income through loss of ownership of the company to the bondholders.

The nature of financial distress costs is that they become increasingly severe as profits get lower. They are some of the most significant costs away from shareholders.

Financial distress costs can be modelled relatively easily. Despite this they are often omitted. A typical omission is the assumption of fixed new business levels. By ignoring the link between profitability, credit rating and new business levels it is possible to ignore one of the most significant risks facing shareholders of an insurance company.

Because of their importance to valuation and the ease of modelling, costs of financial distress are the most obvious item to add to a valuation model.

3.3.5 No agency costs

Agency costs are payments to stakeholders other than shareholders and bondholders resulting from self-serving management decisions. The payments could be explicit or implicit.

Examples of explicit cashflows are remuneration schemes that pay large bonuses when profits are high but still pay a salary when profits are low. There can also be a payment when profits are low if managers leave with a golden handshake.

Implicit cashflows can be more severe for the shareholder because they can also involve large payments to other stakeholders. Managers' pay levels are often correlated to the turnover of the business they manage. This can result in cashflows to customers from loss leading behaviour in an effort to gain market share. It can also lead to the acquisition of ill-suited subsidiaries that destroy value but increase the turnover. There are more opportunities for these agency costs to occur when profits are high.

Agency costs can also occur in financial distress if management act to preserve their reputation rather than acting in the best interests of the shareholders. For example, priorities change towards building personal relationships with future employers and away from the shareholders of the current employer.

Mitigation of agency costs is possible through effective corporate governance procedures. Effective procedures reduce expected agency costs and increase the value of the company.

Agency costs are likely to be a significant cost because of the influence that managers have over the fortunes of a company. As a significant cost they should be included in a cashflow model.

Quantitative modelling of agency costs is not easy. It would therefore be useful to have a high level approach to capture them. Frictional cost functions are a high level solution that can help.

3.3.6 No new information provided by financing decisions

This assumption means that a change in the financing of the company (including dividend announcements) does not change investors expectations of future profit.

This is more difficult to demonstrate because financing decisions are often accompanied by extra information. For example, a company might announce that they are cutting a dividend *because* sales have been falling. There are two pieces of information here, both arriving at the same time. The fall in sales, and the dividend cut. Should the share price fall on such an announcement we might suspect that falling sales was the cause. Unfortunately it would be difficult to demonstrate this conclusively.

This is a difficult assumption to test and relates to optimal management information rather than company valuation. For these reasons we do not consider it further.

3.4 Definition of Frictional Costs

We can plot each of the assumptions on an illustration of *modelling difficulty* vs. *impact on valuation*.

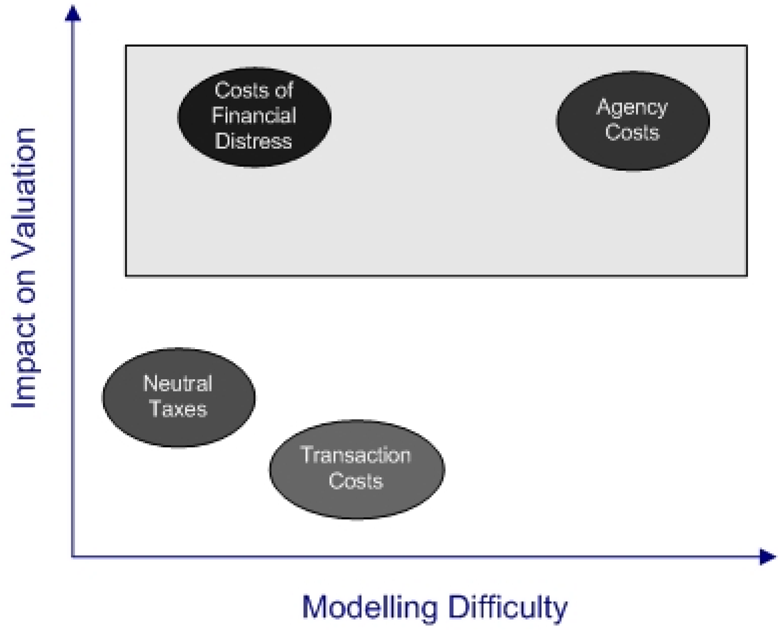

Figure 3-1

The top half of the square (in grey) should be the priority for making cashflow models more realistic. This includes costs of financial distress and agency costs. We conclude that accounting for financial distress costs and agency costs are a promising avenue to close the gap between existing cashflow modelling and market valuation.

The cashflows from both of these sources meet the following criteria. These are:

- Cashflows to other stakeholders at the expense of shareholders and bondholders
- Positive & irrecoverable
- Convex functions of profit

This concurs with our description of frictional costs at the beginning of this paper.

Frictional costs are positive, irrecoverable cashflows away from shareholders and bondholders that have a convex relationship with profit.

4 INTRODUCTION TO FRICTIONAL COST FUNCTIONS

4.1 Introduction

In this section we consider which cashflows are usually included in company modelling. We also introduce our preferred technique for making the cashflow model realistic.

4.2 Existing Modelling

Many costs are explicitly included in cashflow models. Examples include:

- Taxation formulae
- Costs of regulation
- Salary costs
- New business costs
- Claims costs
- Marketing costs

Some of these costs could be counted as frictional costs because of their convex nature. Tax formulae with asymmetries are an obvious example. Higher new business costs due to loss-leading when profits are high, is another.

A prudent management might include less obvious costs in their cashflow model when profits are low. Examples include:

- Capital raising costs
- Bankruptcy costs
- Redundancy costs
- Loss of credit sensitive business

They might also include less obvious costs in their cashflow model when profits are high. Examples include:

- Extra management and staff jollies
- Extra bonus payments
- Temporary staff costs
- Failure to close unprofitable business units
- Lack of financial control
- Unwise mergers and acquisitions

All of these cashflows could be included as frictional costs because they are all positive, irrecoverable, convex cashflows away from the shareholders and bondholders.

Modellers need to trade off between the accuracy, time cost and financial cost of building a company model. Therefore it is unusual to find cashflow models that include all the above. It is also usual for models to include approximate formulae and approximate parameters.

Management influences the choice of which cashflows to include, what formula approximations to use and which parameters to use. If management are optimistic in making these choices, the model will overstate the cashflows payable to shareholders.

It is not necessary for management to be conscious of making optimistic choices. Simplifications and omissions can lead to inadvertent optimism. For example, the assumption of deterministic new business volumes is often made for model simplicity.

Even the least optimistic management would have difficulty in explicitly including all the frictional costs.

Examples of cashflows that are difficult to quantify or model are:

- Changes in regulation or government policy
- Changes in tax regimes
- Employee fraud
- Fire risk
- Business interruption
- Defection of key staff members

4.3 Realistic Cashflow Models

The best efforts of management are still likely to result in a cashflow model that overstates the cashflows to shareholders and bondholders. We use the term *idealistic* to describe this type of cashflow model. The risk-adjusted price of cashflows from an *idealistic* model will overstate the valuation of the company.

The cashflow model will be *realistic* when the risk-adjusted price of the cashflows from the model reconcile to the market value of the company. Ideally we would like to make the model realistic using a technique that still made the model useful for analysis.

To be useful for analysis any technique will have to conform to a set of rules. The rules ensure that the model will still be able to make meaningful comparisons. This is necessary to maintain its usefulness as a decision making tool. The rules are known are risk measure axioms, and are introduced in the next section.

4.4 Making the Cashflow Model Realistic

Our proposed solution is to aggregate all of the missing cashflows, approximations and optimism into a *frictional cost function.* The frictional cost function is used to calculate a stochastic cashflow. This stochastic cashflow is deducted from the *idealistic profit* to give the *realistic profit*.

Our review of Modigliani & Miller suggested that frictional costs have a significant influence on company valuation. Therefore, we assume that the frictional costs dominate the functional form of the frictional cost function. We choose a frictional cost function that reflects the relationship between frictional costs and our choice of independent variable.

It makes sense to choose the most significant driver of the aggregate frictional costs as the independent variable. For simplicity we choose one variable. Usually we choose the independent variable to be profit, although other choices are possible.

Aggregate frictional costs are the combination of financial distress costs and agency costs. The nature of these costs suggests that aggregate frictional costs should rise steeply when profit is lower than expected, and rise more gently when profit is higher than expected. We therefore propose a frictional cost function with the form of the graph below.

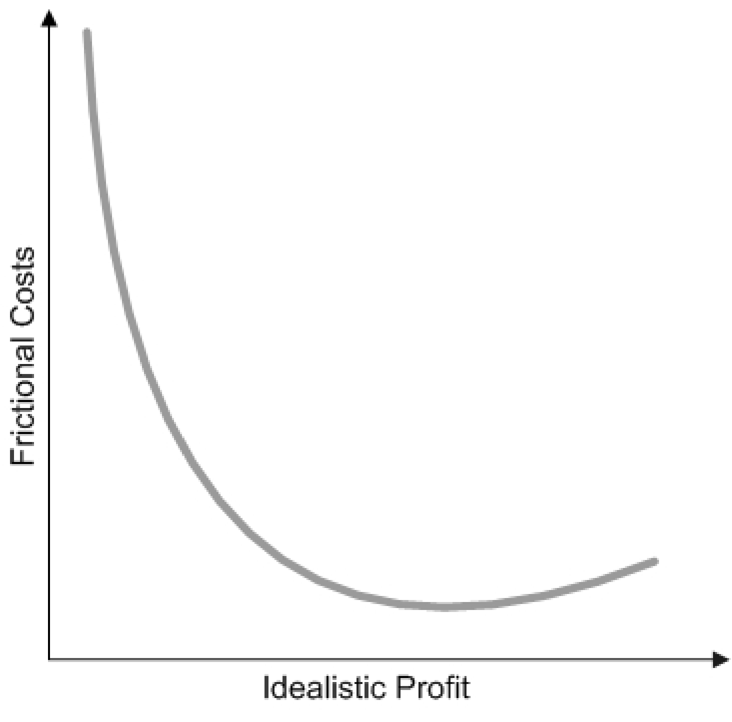

Figure 4-1

5 INTRODUCTION TO RISK MEASURES

5.1 Introduction

In this section we introduce the concept of a risk measure. This is based on the insurance concept of a Coherent Risk Measure as described in Artzner (1999).

We begin this section with a definition of a risk measure.

We then introduce two conventions for specifying a risk measure. Some risk measure axioms differ slightly depending on whether the risk measure describes the *risk-adjusted price* or a value for making an *adjustment* to a *non-risk-adjusted price*. The reason for the two conventions is historical rather than any fundamental difference. If the literature were written again only one of these conventions would be chosen.

We then explain the reason for each risk measure axiom. We tabulate the differences between its definition under the *price* and *adjustment* conventions. We also tabulate the differences for finance pricing and insurance pricing.

Finally we illustrate an example a risk measure from insurance pricing followed by an example from finance pricing.

5.2 Definition of Risk Measure

We now define the concept of a risk measure.

A risk measure satisfies a prescribed set of axioms that are required for meaningful comparisons to be made between risky cashflows.

5.3 Two Types of Risk Measure

5.3.1 Price Risk Measure

A *price* risk measure is a price for a risky cashflow that has already been adjusted to account for the risk of the cashflow.

A risk-adjusted price calculated using deflators is an example of a price risk measure, where X is a risky cashflow and D is the deflator.

$$\rho(X) = E[DX]$$

Equation 5-1

5.3.2 Adjustment Risk Measure

An *adjustment* risk measure is a value which represents the risk of a risky cashflow. It can be used to make a risk adjustment to a non risk-adjusted price.

Volatility is an example of an adjustment risk measure from insurance. The volatility could be used to calculate an adjustment to the risk discount rate using an efficient frontier. This can be expressed as $\rho(X)$ where X is a risky cashflow.

$$\rho(X) = \sigma[X]$$

Equation 5-2

5.4 Risk Measure Axioms

5.4.1 Translation Invariance

Translation invariance ensures that the price measure only adjusts prices for risk. It does this by ensuring that the addition of a risk-free cashflow does not change the measurement of risk.

The axiom is summarised for the four cases below where, λ is a risk-free amount of cash, X is a risky cashflow, and r is the risk-free rate.

	Insurance	***Finance***
Price	$\rho(X+\lambda)=\rho(X)+\frac{1}{1+r}\lambda$	$\rho(X+\lambda)=\rho(X)+\frac{1}{1+r}\lambda$
Adjustment	$\rho(X+\lambda)=\rho(X)$	$\rho(X+\lambda)=\rho(X)$

Table 5-1

We notice that there is no difference between finance and insurance, but the *price* risk measure differs slightly from the *adjustment* risk measure because the price of an asset needs to change for a risk-free cashflow.

5.4.2 Positive Homogeneity

Positive homogeneity ensures that meaningful comparisons can be made between risks of different size. It does this by ensuring that scaling all the cashflows by a constant factor increases the risk measure by the same constant factor.

The axiom is summarised for the four cases below, where λ is a scaling factor and X is a risky cashflow.

	Insurance	***Finance***
Price	$\rho(\lambda X)=\lambda\rho(X)$	$\rho(\lambda X)=\lambda\rho(X)$
Adjustment	$\rho(\lambda X)=\lambda\rho(X)$	$\rho(\lambda X)=\lambda\rho(X)$

Table 5-2

We notice that there is no difference between any of the cases.

5.4.3 Subadditivity

Subadditivity is the most interesting axiom we consider.

Subadditivity captures the effect of an insurance contract being considered less risky when it is part of a portfolio. Equivalently, it captures the lower value of reserves required for holding an insurance contract in a portfolio of similar contracts. Subadditivity for financial contracts has a slightly different definition which captures arbitrage-free pricing.

The axiom is summarised for the four cases below where X_1 & X_2 are two risky cashflows.

	Insurance	***Finance***
Price	$\rho(X_1+X_2)\geq\rho(X_1)+\rho(X_2)$	$\rho(X_1+X_2)=\rho(X_1)+\rho(X_2)$
Adjustment	$\rho(X_1+X_2)\leq\rho(X_1)+\rho(X_2)$	$\rho(X_1+X_2)=\rho(X_1)+\rho(X_2)$

Table 5-3

We see that the *price* risk measure differs from the *adjustment* risk measure in the direction of the inequality. We need to be careful with signs because negative cashflows will cause the inequality to be reversed. We have used the shareholders' perspective as our convention.

The significant difference is the equality required for finance pricing and the inequality required for insurance pricing. This is discussed in more detail at the end of this section.

5.4.4 Monotonicity

Monotonicity ensures that if the cashflows from one risk, X_A, always exceed another, X_B, the price of X_A always exceeds the price of X_B.

This axiom ensures that an insurance policy that always pays out more than another will always require higher reserves. Equivalently, it ensures that an asset that always pays more than another will always have a higher price.

We can summarise this in the following table where X_A and X_B are risky cashflows. $X_A(s)$ and $X_B(s)$ are the cashflows X_A and X_B in state s. The symbol $\forall$ means *for all*.

	Insurance	***Finance***
Price	$X_A(s)<X_B(s)\forall s$ $\Rightarrow\rho(X_A)<\rho(X_B)$	$X_A(s)<X_B(s)\forall s$ $\Rightarrow\rho(X_A)<\rho(X_B)$
Adjustment	*N/A*	*N/A*

Table 5-4

We see that the axiom is only relevant for a *price* risk measure. Otherwise there is no difference between finance and insurance pricing.

5.5 Insurance Example

5.5.1 Introduction

We now demonstrate a simple price risk measure that satisfies all the insurance axioms listed above. The risk measure is the maximum loss incurred by the insurer. Because our convention is from the shareholders' perspective, losses are negative. The maximum loss is therefore expressed as follows.

$$\rho(X)=Min(X)$$

Equation 5-3

Consider two insurance companies, Company A and Company B, with associated losses of X_A and X_B. In the table below, we summarise two possible scenarios for losses incurred by each company. In this example we assume the risk-free rate is zero for simplicity.

Scenario	X_A	X_B
1	-1.00	-3.00
2	-2.00	-2.00
$\rho(\mathbf{X})$	-2.00	-3.00

Table 5-5

We now work through each risk measure to show that it is satisfied.

5.5.2 Translation Invariance

Consider increasing the loss by 1. The scenarios have now changed as follows:

Scenarios	X_A	X_A - 1
1	-1	-1+-1 = -2
2	-2	-3
$\rho(\mathbf{X})$	-2	-3

Table 5-6

We see that the risk measure for $(X_A\text{-}1)$ is equal to the risk measure of X_A less one. Remembering that the risk-free rate is zero in this example, we have demonstrated the translation invariance axiom is satisfied as follows.

$$\rho(X_A - 1) = \rho(X_A) - 1$$

Equation 5-4

5.5.3 Positive Homogeneity

Consider scaling the loss amount by a constant factor of 2. The scenarios have now changed as follows.

Scenarios	X_A	$2X_A$
1	-1	2×-1 =-2
2	-2	-4
$\rho(X)$	-2	-4

Table 5-7

We see that the risk measure for ($2\ X_A$) is twice the risk measure for X_A. Therefore we have demonstrated the positive homogeneity axiom is satisfied as follows.

$$\rho(2 \cdot X_A) = 2 \cdot \rho(X_A)$$

Equation 5-5

5.5.4 Subadditivity

Assume that Company A and Company B have now merged to form two divisions of Company C.

Scenarios	X_A	X_B	$X_C = X_A + X_B$
1	-1	-3	-1+-3 = -4
2	-2	-2	-4
ρ(X)	-2	-3	-4

Table 5-8

We see that the maximum loss incurred by Company C is –4. This is lower than the sum of the maximum loss of the individual divisions, -5. Therefore, we have demonstrated the subadditivity axiom is satisfied as follows.

$$\rho(X_A)+\rho(X_B)\leq\rho(X_C)=\rho(X_A+X_B)$$

Equation 5-6

5.5.5 Monotonicity

Consider the cashflows from each division.

Scenarios	X_A	X_B
1	-1	-3
2	-2	-2
ρ(X)	-2	-3

Table 5-9

We notice the losses from Division B always exceed Division A. We also notice that the risk measure of Division A is bigger than Division B. Therefore we have demonstrated the monotonicity axiom is satisfied as follows.

$$X_A(s)\geq X_B(s)\forall s\Rightarrow\rho(X_A)\geq\rho(X_B)$$

Equation 5-7

5.6 Finance Example

5.6.1 Introduction

In this sub-section we demonstrate that deflator valuation obeys the *price* risk measure axioms for financial assets. We define the risk measure as follows.

$$\rho(X) = E[DX]$$

Equation 5-8

5.6.2 Translation Invariance

We consider the addition of a risk-free amount of cash, λ, to a risky cashflow, *X*.

After the addition of the risk-free cash the risk measure would change according to the following expression.

$$\rho(X + \lambda) = E[DX + D\lambda] = E[DX] + E[D]\lambda = \rho(X) + \frac{1}{1+r}\lambda$$

Equation 5-9

We see that the *price* risk measure for translation invariance holds for deflator pricing.

5.6.3 Positive Homogeneity

We consider scaling the payments by a constant factor, λ. After the payments have been scaled we can take the factor outside the expectation as follows.

$$\rho(\lambda X) = E[\lambda DX] = \lambda E[DX] = \lambda \rho(X)$$

Equation 5-10

We can see that deflator valuation satisfies the positive homogeneity axiom.

5.6.4 Subadditivity

We consider the addition of two risky cashflows, X_A and X_B. After the payments have been added we can multiply out the expression.

$$\rho(X_A + X_B) = E[D(X_A + X_B)] = E[DX_A] + E[DX_B] = \rho(X_A) + \rho(X_B)$$

Equation 5-11

We see that the deflator valuation satisfies the equality subadditivity axiom. If it did not then there would be an arbitrage opportunity.

5.6.5 Monotonicity

We consider the price of two assets, X_A and X_B, valued using deflators. The price of Asset X_A is given by the following expression. We assume there are only two future states and use brackets to indicate in which state the cashflow occurs.

$$\rho(X_A) = E[DX_A] = D(1)X_A(1) + D(2)X_A(2)$$

Equation 5-12

Similarly, the price of Asset X_B is given by the following expression.

$$\rho(X_B) = E[DX_B] = D(1)X_B(1) + D(2)X_B(2)$$

Equation 5-13

If the cashflows of Asset X_A are always greater than Asset X_B we can write the following expressions.

$$X_A(1) - X_B(1) > 0 \;:\; X_B(2) - X_A(2) < 0$$

Equation 5-14

Now we can see if this means that the price of Asset X_A always exceeds Asset X_B. We set up the following expression to test.

$$\rho(X_A) > \rho(X_B) \Rightarrow D(1)X_A(1) + D(2)X_A(2) > D(1)X_B(1) + D(2)X_B(2)$$

Equation 5-15

We can rearrange the expression above as follows.

$$D(1)(X_A(1) - X_B(1)) > D(2)(X_B(2) - X_A(2))$$

Equation 5-16

Using the inequalities above and the fact that deflators are always positive, we see that Equation 15-6 is always true. Therefore the axiom of monotonicity holds for deflator valuation.

5.7 Discussion

Can we understand why insurance pricing axioms should differ from financial pricing axioms when they both price future cashflows?

The difference between the axioms is the subadditivity expression which requires equality for finance pricing but only inequality for insurance pricing.

The reason for the equality in finance pricing is that an inequality would permit an arbitrage. Does this mean that arbitrage exists in insurance pricing and if so why does no-one exploit it until it disappears as occurs in financial markets? We need to understand the difference in the markets.

Financial markets are characterised by high levels of securitisation (described in Appendix B), low dealing costs and large liquid markets. Financial assets traded in these markets are characterised by low costs of ownership that scale linearly with the amount held. This means that two financial assets are worth no more together in a portfolio than they are apart.

Insurance markets are characterised by low levels of securitisation, high dealing costs and illiquid markets. Insurance assets traded in these markets are characterised by high costs of ownership that reduce non-linearly the more policies that are written. The high costs of ownership include regulatory capital, moral hazard, claims costs and sales costs. These costs can often be spread across many polices which results in a cost of ownership that reduces non-linearly with the number of insurance policies written.

We would argue that in the absence of these costs, insurance pricing would be the same as financial pricing. Each of these costs is a payment to one of the other stakeholders in the company. We believe that inclusion of these costs in the cashflow model would have two benefits.

- It would bring insurance pricing into line with financial pricing.
- It would focus attention on the true causes of value creation through diversification and help improve the efficiency of insurance companies.

6 A FRICTIONAL COST RISK MEASURE

6.1 Introduction

In this section we start by considering which of the risk measure axioms are relevant for a frictional cost risk measure.

We then introduce a simple risk measure for frictional costs similar to the deflator risk measure illustrated in the previous section. We demonstrate how it satisfies the set of risk measure axioms we have chosen.

We finish by introducing a more realistic frictional cost function that can be substituted for the simpler version.

6.2 Frictional Cost Axioms

We would like our frictional cost function to allow meaningful comparisons to be made between the contribution to frictional costs of different business units. We therefore specify a set of axioms that we would like to see in a risk measure for frictional costs.

6.2.1 Translation Invariance

An immediate risk-free cashflow should not affect the level of frictional costs, just increase the value of the company by the risk-free amount. We therefore include translation invariance with the following axiom.

$$\rho\left(X+\lambda\right)=\rho\left(X\right)$$

Equation 6-1

6.2.2 Positive Homogeneity

We would like to make meaningful comparisons between business units of differing sizes. Therefore scaling the cashflow by a constant factor should scale the frictional costs by the same constant factor. We therefore include positive homogeneity with the following axiom.

$$\rho\left(\lambda X\right)=\lambda\rho\left(X\right)$$

Equation 6-2

6.2.3 Subadditivity

Under some circumstances we would like the risk measure to recognise value creation through diversification. This would enable us to see the diversification benefit of grouping several business units into a single company. This would mean including subadditivity with the following axiom.

$$\rho\left(X_A+X_B\right)\leq\rho\left(X_A\right)+\rho\left(X_B\right)$$

Equation 6-3

It would be also be useful to have a risk measure that recognised arbitrage-free pricing. This would enable us to calculate a market consistent value for each business unit that summed to the market value of the company. This would mean including subadditivity with the following axiom.

$$\rho(X_A + X_B) = \rho(X_A) + \rho(X_B)$$

Equation 6-4

Ideally we would like a risk measure that could be adapted to either of these subadditivity axioms. This turns out to be possible.

6.2.4 Monotonicity

Consider two assets for which the cashflows from X_A always exceed X_B. Because the frictional costs are a convex function of the cashflow, the expected value of the frictional costs for X_A could be greater or less than the frictional costs for X_B.

For example consider the convex function, $f(x)=x^2$. Also consider cashflows from X_A being in the range {-1,+1} and cashflows from X_B are in the range {-3,-2}. The cashflows from X_A are always greater than X_B, but $f(X_A)$ is always less than $f(X_B)$.

Therefore the monotonicity axiom is not appropriate for a frictional cost risk measure.

6.3 Frictional Cost Risk Measure

We now introduce the frictional cost risk measure. We define it as the expected value of a frictional cost function, $\theta(P)$, of the *idealistic profit, P*.

$$\rho(P) = E[\theta(P)]$$

Equation 6-5

The frictional cost function, $\theta(P)$, is defined as:

$$\theta(P) = K\left(\alpha + \frac{1}{\alpha}(P - \beta)^2\right)$$

Equation 6-6

We need to calculate the derivative of the frictional cost function later so it is given here for ease of reference.

$$\frac{\partial\theta(P)}{\partial P} = \frac{2K}{\alpha}(P - \beta)$$

Equation 6-7

The parameter, K, represents the severity of the frictional costs. This is the calibration parameter that is chosen to ensure that there are just enough frictional cost deductions to reconcile the valuation to the market price. For simplicity we will assume that K has a constant value for all examples in the rest of the paper.

$$K = 0.3$$

Equation 6-8

The other parameters are chosen to minimise the expected value of frictional costs for a given value of K. This is necessary for the frictional cost function to satisfy the axioms we have chosen. The appropriate values are as follows.

$$\beta = E[P]$$

Equation 6-9

$$\alpha = \sqrt{E[P^2] - E[P]^2} = \sqrt{VAR[P]} = \sigma[P]$$

Equation 6-10

It is important to understand that the definitions of α and β are integral to the definition of the frictional cost function.

This risk measure will satisfy all the axioms except the strict equality subadditivity axiom required for consistency with financial pricing. When we require this axiom to be satisfied we need to use the *marginal* risk measure to account for the marginal frictional cost allocation of the business unit to the company.

For example consider a company, M, with two business units A and B. The *marginal* risk measure for A is defined as follows.

$$\rho(A) = E\left[A \left. \frac{\partial \theta(M)}{\partial M} \right|_{M=A+B} \right]$$

Equation 6-11

The sum of the marginal risk measure over all business units reconciles to the risk measure for the entire company. This can be expressed as follows.

$$E\left[A \left. \frac{\partial \theta(M)}{\partial M} \right|_{M=A+B} \right] + E\left[B \left. \frac{\partial \theta(M)}{\partial M} \right|_{M=A+B} \right] = E[\theta(M)]$$

Equation 6-12

The mathematical properties of the frictional cost function makes this possible. Frictional cost functions that have this property form part of a special family of functions. The details can be found in Appendix E.

6.4 Adding Deflators to the Risk Measure

We have seen that the deflator risk measure provides a risk adjustment for systematic risk. We can interpret the risk measure for frictional costs as a risk adjustment for frictional costs.

It would simplify the modelling process if we could risk-adjust for systematic risk and frictional costs in one expression. This turns out to be possible because the deflators are always positive and can be included in the risk measure as follows.

$$\rho(P) = E[D\theta(P)]$$

Equation 6-13

The corresponding *marginal* risk measure includes a deflator.

$$\rho(A) = E\left[DA \frac{\partial\theta(M)}{\partial M}\bigg|_{M=A+B} \right]$$

Equation 6-14

The equations for the parameters α and β also need to be amended to include the deflator.

$$\beta = \frac{E[DP]}{E[D]}$$

Equation 6-15

$$\alpha = \frac{1}{E[D]}\sqrt{E[DP^2]E[D] - E[DP]^2}$$

Equation 6-16

We will use the risk measure including deflators because we wish to take account of the systematic risk as well as the frictional costs in our pricing.

6.5 Demonstrating Frictional Cost Axioms

6.5.1 Translation Invariance

We demonstrate that translation invariance is satisfied by the frictional cost risk measure using Asset A from the Feast, Fine & Famine model. We need to ensure that we use the appropriate α and β parameters for X and $X+\lambda$.

First we calculate the constituents of the formulae for α and β. This is done explicitly in the table below.

X	***Feast***	***Fair***	***Famine***	***E[X]***
DA	0.9	1.35	4.05	2.10
DA²	1.35	2.025	12.15	5.18

Table 6-1

Using the values in the table above we can calculate the correct values of α and β for Asset A.

α	β
0.749	2.21

Table 6-2

With the values of α and β calculated we can calculate the present value of the frictional cost.

E[Dθ(A)]	0.427

Table 6-3

Next we rework the calculations after adding a risk-free amount of cash to the payments. The new payments now look as follows.

X	***Feast***	***Fair***	***Famine***	***E[X]***
A+2	1.5+2 = 3.5	3.5	5	4

Table 6-4

We need to recalculate the values of α and β in accordance with the definition of our frictional cost risk measure. Following the same procedure as above we get the following.

X	***Feast***	***Fair***	***Famine***	***E[X]***
D(A+2)	2.1	3.15	6.75	4.00
D(A+2)²	7.35	11.025	33.75	17.38

Table 6-5

Using the values in the table above we can calculate the correct values of α and β for Asset A with the risk-free amount:

α	β
0.749	4.21

Table 6-6

Next we calculate the frictional cost risk measure for Asset A plus the risk-free amount.

E[Dθ(A+2)]	0.427

Table 6-7

Finally we summarise the results below.

E[Dθ(A)]	***E[Dθ(A+2)]***
0.427	0.427

Table 6-8

We see that the frictional cost risk measure has not changed and satisfies the translation invariance axiom we specified.

6.5.2 Positive Homogeneity

We demonstrate that positive homogeneity is satisfied by the frictional cost risk measure using Asset A from the Feast, Fine & Famine model. We have already calculated the frictional cost risk measure for Asset A, so it only remains to calculate the frictional cost risk measure for a multiple of the Asset A cashflows. We choose a multiple of 2.

X	***Feast***	***Fair***	***Famine***	***E[X]***
2A	2×1.5 = 3	3	6	4

Table 6-9

We need to recalculate the α and β parameters. The intermediate calculations now become:

X	*Feast*	*Fair*	*Famine*	*E[X]*
D(2A)	1.8	2.7	8.1	4.20
$D(2A)^2$	5.4	8.1	48.6	20.70

Table 6-10

Next we can calculate the new values for α and β.

α	β
1.50	4.42

Table 6-11

Now we calculate the frictional cost risk measure.

E[Dθ(2A)]	0.854

Table 6-12

Finally we summarise the results below.

E[Dθ(A)]	*2E[Dθ(A)]*	*E[Dθ(2A)]*
0.427	0.854	0.854

Table 6-13

We see that the frictional cost risk measure has increased by a factor of 2 showing that positive homogeneity axiom we specified is satisfied.

6.5.3 Subadditivity

6.5.3.1 Insurance Pricing

We now demonstrate that the inequality subadditivity axiom is satisfied by the frictional cost risk measure using Division A, Division B and Company M from the Feast, Fine & Famine model. We use the ordinary frictional cost risk measure so we satisfy the inequality version of the subadditivity axiom.

We have already calculated the risk measure for Division A. It remains to calculate the risk measure for Division B, and Company M.

Following the same procedure as before we present the intermediate calculations for Division B.

X	*Feast*	*Fair*	*Famine*	*E[X]*
DB	1.8	1.8	1.35	1.65
DB^2	5.4	3.6	1.35	3.45

Table 6-14

Now we calculate α and β for Division B.

α	β
0.784	1.74

Table 6-15

Next we calculate the frictional cost risk measure for Division B.

E[Dθ(B)]	0.447

Table 6-16

Following the same procedure as before we can calculate the intermediate variables for Company M.

X	***Feast***	***Fair***	***Famine***	***E[X]***
DM	2.7	3.15	5.4	3.75
DM^2	12.15	11.025	21.6	14.93

Table 6-17

Now we calculate α and β for Company M.

α	β
0.359	3.95

Table 6-18

Next we calculate the frictional cost risk measure for Company M.

E[Dθ(M)]	0.205

Table 6-19

Finally, we compare the sum of the frictional cost risk measures for Division A and Division B separately with the frictional cost risk measure for the portfolio of Division A & Division B, i.e. Company M.

E[Dθ(A)]	***E[Dθ(B)]***	***E[Dθ(A)]+ E[Dθ(B)]***	***E[Dθ(M)] = E[Dθ(A+B)]***
0.427	0.447	0.874	0.205

Table 6-20

We see that the sum of risk measures calculated separately is higher than when calculated together. Therefore we have demonstrated that the inequality version of the subadditivity axiom is satisfied by the frictional cost risk measure.

We can also calculate the diversification benefit from Division A and Division B being two divisions of Company M.

E[Dθ(A)]+ E[Dθ(B)]-E[Dθ(A+B)]	0.874-0.205 = 0.669

Table 6-21

We see there is a substantial diversification benefit. This is the result of a negative correlation between the cashflows of Division A and Division B.

6.5.3.2 Financial Pricing

We now demonstrate that subadditivity with a strict equality is satisfied by the *marginal* risk measure. We illustrate this using Division A, Division B and Company M from the Feast, Fine & Famine model.

We start by calculating the derivative of the Company M frictional cost function. We can reuse the α and β parameters calculated above for Company M.

X	***Feast***	***Fine***	***Famine***	***E[X]***
M	4.5	3.5	4	4
$\frac{\partial\theta(M)}{\partial M}$	0.924	-0.748	0.0880	0.0880

Table 6-22

Next we calculate the intermediate stages for Division A and Division B.

X	***Feast***	***Fine***	***Famine***	***E[X]***
$D \cdot A \cdot \frac{\partial\theta(M)}{\partial M}$	0.831	-1.01	0.356	0.0594
$D \cdot B \cdot \frac{\partial\theta(M)}{\partial M}$	1.663	-1.35	0.119	0.145

Table 6-23

Finally we summarise the calculations.

$E[D \cdot A \cdot \frac{\partial\theta(M)}{\partial M}]$	$E[D \cdot B \cdot \frac{\partial\theta(M)}{\partial M}]$	$E[D \cdot A \cdot \frac{\partial\theta(M)}{\partial M}]+[D \cdot B \cdot \frac{\partial\theta(M)}{\partial M}]$	$E[D \cdot \theta(M)]$
0.0594	0.145	0.0594+0.145 = 0.205	0.205

Table 6-24

We see that the *marginal* risk measure for Division A and Division B sum to the risk measure for Company M.

6.6 A More Sophisticated Frictional Cost Function

The frictional cost function we have presented above is a symmetrical function of profit. While it is relatively easy to work with, it does not fit with our requirement to capture the more severe frictional costs for low profitability. A more realistic risk measure involves using the following frictional cost function.

$$\theta(x) = \int_{-\infty}^{x} \{\gamma[G(u)] - G(u)\gamma'[G(u)]\}du + \int_{x}^{\infty} \{\gamma[G(u)] + [1-G(u)]\gamma'[G(u)] - 1\}du$$

Equation 6-17

In the expression above γ is a non-decreasing concave function on [0,1], with *γ(0)* = 0 and *γ(1)* = 1. *G* is a non-decreasing right continuous function on [-∞, ∞] with *G*(-∞) = 0 and *G*(∞)=1. *G* is chosen to minimise the expected value of frictional costs. This function is described in detail in Smith, Moran, Walczack (2003).

This frictional cost function satisfies the frictional cost axioms we have presented in this section. The mechanics are similar. The only difference is that the algebra is more difficult and more complex techniques are required to do calculations.

This is the functional form used in to value a General Insurance company in Christofides & Smith (2001). Illustrations of the function in Equation 6-17 can be found in their paper.

7 FRICTIONAL COST PLAYGROUND

7.1 Introduction

In this section we explore some of the implications of using the frictional cost risk measure described in the last section. Using the Feast, Fine & Famine model we illustrate the following

- Allocation of frictional costs to business units
- Equivalent risk premium adjustments

7.2 Capitalisation-Weighted Frictional Cost Allocation

In the last section we saw that using the marginal risk measure we could allocate the frictional costs to each line of business so they summed to frictional costs at company level.

The result is summarised in the following table.

$E[D\cdot A\cdot \frac{\partial\theta(M)}{\partial M}]$	$E[D\cdot B\cdot \frac{\partial\theta(M)}{\partial M}]$	$E[D\cdot A\cdot \frac{\partial\theta(M)}{\partial M}]+[D\cdot B\cdot \frac{\partial\theta(M)}{\partial M}]$	$E[D\cdot\theta(M)]$
0.0594	0.145	0.0594+0.145 = 0.205	0.205

Table 7-1

A different approach to the allocation of the frictional costs could have been to use a capitalisation-weighted basis. We can do this and see if the results are the same.

First we calculate the percentage of the company's capitalisation that is attributed to Division A and Division B and allocate the frictional costs calculated at company level using these percentages.

	Capitalisation	*Frictional Cost Allocation*
Division A	2.1/3.75 = 0.56	0.115
Division B	0.44	0.090

Table 7-2

Now we compare the capitalisation-weighted allocation with the marginal allocation using a frictional cost risk measure.

	Capitalisation Allocation	*Marginal Allocation*
Division A	0.115	0.0594
Division B	0.0900	0.145

Table 7-3

We see that they are not the same. Using the frictional cost risk measure Division B is bearing a larger share of the frictional costs than Division A. This is because Division B has less diversification benefit relative to Division A. This should make sense to us because Division B makes the company profit more risky than Division A. Capitalisation-weighted allocation does not take account of the relative diversification benefit of the two Divisions.

7.3 Equivalent Discount Rates

The equivalent discount rate is the constant rate of interest required to give the risk-adjusted valuation. The equivalent discount rate, r_X, for a risky cashflow, X, is defined as follows.

$$r_X = \frac{E[X]}{E[DX]} - 1$$

Equation 7-1

We now consider what adjustment would need to be made to the equivalent risk discount rate if we were not to deduct frictional costs from the profit.

First we tabulate the expected values of the frictional costs, $E[\theta(P)]$, the *idealistic* profit, $E[P]$, and the *realistic* profit, $E[P-\theta(P)]$, for Company M. We also tabulate the market-consistent value of Company M after the deduction of the frictional costs, $E[D(P-\theta(P))]$.

P	***E[P]***	***E[θ(P)]***	***E[P-θ(P)]***	***E[D(P-θ(P))]***
Company M	4	0.249	3.75	3.54

Table 7-4

Now we calculate the equivalent discount rate that would need to be applied to *idealistic* profit to get the market consistent value. We also calculate the equivalent discount rate that would need to be applied to *realistic* profit to get the market consistent value. The difference is the extra discount rate adjustment needed to explain the market valuation.

	Discount Rate Using Idealistic Profit	***Discount Rate Using Realistic Profit***	***Extra Discount Rate***
Company M	0.128	0.0579	0.0703

Table 7-5

We see that if we were using an *idealistic* cashflow model we would have had to adjust the discount rate upwards to account for the frictional costs. This additional discount rate is attributed to unsystematic risk by some methodologies. We have achieved the same result by considering what is missing from the cashflow model.

Because we can allocate frictional costs to each division consistent with financial pricing, we can calculate the equivalent discount rate at division level.

First we tabulate the marginal allocation of frictional costs, $E[X\theta'(P)]$, the *idealistic* profit, $E[X]$, and the *realistic* profit, $E[X-X\theta'(P)]$, for Division A and Division B. We also tabulate the market-consistent values of Division A and Division B after the deduction of the frictional costs, $E[D(X-X\theta'(P))]$.

X	***E[X]***	***E[Xθ'(P)]***	***E[X-Xθ'(P)]***	***E[D(X-Xθ'(P))]***
Division A	2	0.176	1.82	2.04
Division B	2	0.455	1.55	1.50

Table 7-6

Next we calculate the equivalent discount rate that would need to be applied to the expected value of *idealistic* profit to get the market-consistent value. We also calculate the equivalent discount rate that would need to be applied to the expected value of *realistic* profit to get the market-consistent value. The difference is the additional discount rate.

	Discount Rate Using Idealistic Profit	***Discount Rate Using Realistic Profit***	***Extra Discount Rate***
Division A	-0.0199	-0.106	0.0862
Division B	0.329	0.0270	0.302

Table 7-7

We see that if we were using a *idealistic* cashflow model we would have had to adjust the discount rate upwards on Division A and Division B to account for the frictional costs. However a significantly higher adjustment is required for Division B reflecting its greater contribution to the frictional costs of Company M.

8 UNSYSTEMATIC RISK & VALUATION

8.1 Introduction

Unsystematic risks are discounted at the risk-free rate. However, this does not mean that the level of unsystematic risk does not influence the risk-adjusted valuation. Increasing variability in the profits due to unsystematic risk does affect valuation by increasing the expected value of frictional costs.

Another smaller effect can occur with unsystematic risks. After having frictional costs deducted the cashflows can become correlated with the deflator. This means they acquire a small amount of systematic risk. A small risk premium, which could be negative, is required.

In this section we use the Feast, Fine & Famine model to illustrate how unsystematic risks interact with frictional costs to influence the valuation.

8.2 Feast, Fine & Famine Example

First we introduce a new asset to the model, Asset U. Asset U is uncorrelated with the deflator and therefore qualifies as an unsystematic risk.

X	***Feast***	***Fine***	***Famine***	***E[X]***
Asset U	2	4	2.25	2.75

Table 8-1

We can calculate its price using the deflators from the Feast, Fine & Famine model.

X	***Feast***	***Fine***	***Famine***	***E[X]***
DU	1.2	3.6	3.0375	2.61

Table 8-2

Now we can calculate the state dependent returns for Asset U:

R(X)	***Feast***	***Fine***	***Famine***	***E[R(X)]***
Asset U	-0.234	0.531	-0.139	0.0526

Table 8-3

Immediately we spot that Asset U's expected return is equal to the risk-free rate which is consistent with it being an unsystematic risk. We can prove Asset U is an unsystematic risk by evaluating the covariance between returns on Asset U and the deflator. This is done in the following table.

R(X)	***COV(R(X),D) = E[R(X),D] - E[R(X)]E[D]***
Asset U	0.333(-0.234x0.6+0.531x0.9x-0.139x1.35)-0.95x0.05236 = 0

Table 8-4

We now calculate the frictional costs associated with this asset. Following the usual procedure, using $K = 0.3$, we calculate the values of α and β.

α	β
0.855	2.75

Table 8-5

Now we can calculate the frictional costs in each state.

X	***Feast***	***Fine***	***Famine***	***E[X]***
$\theta(U)$	0.454	0.805	0.344	0.534

Table 8-6

We see that frictional costs are incurred from unsystematic risk in the profit.

Next we deduct the frictional costs from the *idealistic* profit to get the *realistic* profit.

X	***Feast***	***Fine***	***Famine***	***E[X]***
$U-\theta(U)$	1.55	3.20	1.91	2.22

Table 8-7

We see that the expected value of the cashflows from Asset U has fallen from 2.75 to 2.22.

Using the deflators we can calculate the risk-adjusted value of Asset U less frictional costs.

X	***Feast***	***Fine***	***Famine***	***E[X]***
$D(U-\theta(U))$	0.928	2.88	2.57	2.13

Table 8-8

Finally we look at the return implied by deducting the frictional costs.

R(X)	***Feast***	***Fine***	***Famine***	***E[R(X)]***
$U-\theta(U)$	-0.273	0.503	-0.103	0.0425

Table 8-9

We see that the return on the asset has fallen slightly as a result of the frictional cost deduction. This is because the convexity of the frictional cost function has caused Asset U returns to become slightly correlated with the deflator. We can show this by calculating the covariance of returns on Asset U, less frictional costs, with the deflator.

R(X)	***COV(R(X),D) = E[R(X),D] - E[R(X)]E[D]***
$U-\theta(U)$	0.333(-0.273x0.6+0.503x0.9x-0.103x1.35)-0.0425x0.95 = 0.00964

Table 8-10

Therefore frictional costs have two effects on unsystematic risks.

1. A reduction of the expected value of the cashflows.
2. A change to the equivalent discount rate.

8.3 Expected Value of Frictional Costs vs. Unsystematic Risk

We can examine how the expected value of frictional costs changes as the unsystematic risk increases. We illustrate this by multiplying the cashflows of Asset U by 2. This gives us a new risky asset with twice the volatility of Asset U.

The revised cashflows are shown in the following table.

X	*Feast*	*Fine*	*Famine*	*E[X]*
2U	4	8	4.5	5.5

Table 8-11

If we repeat the calculation shown above for Asset U we can find the revised value for the expected frictional cost. We omit the detailed calculations for brevity and present the results in the following table.

X	*Feast*	*Fine*	*Famine*	*E[X]*
$\theta(2U)$	0.908	1.61	0.688	1.07

Table 8-12

We can see that the expected value of the frictional costs has increased with the volatility of the cashflows. This is a consequence of the convex nature of the frictional cost function.

8.4 Systematic Division & Unsystematic Division

Consider Company N that consists of Division U and Division B. The cashflows are tabulated below.

X	*Feast*	*Fine*	*Famine*	*E[X]*
Division U	2	4	2.25	2.75
Division B	3	2	1	2
Company N	5	6	3.25	4.75

Table 8-13

We repeat the calculations set out above, using the *marginal* risk measure to allocate frictional costs of Company N to Division U and Division B. For brevity we do not include all the intermediate working but present the attribution of frictional costs to the two divisions of the company.

$E[DU\theta'(N)]$	$E[DB\theta'(N)]$	$E[D\theta(N)]$
0.376	0.323	0.699

Table 8-14

We can see that that both the systematic risk division, B, and the unsystematic risk division, U, bear a significant proportion of the frictional costs.

8.5 Discussion

All the effects illustrated in this section are automatically taken into account by using the deflator and frictional cost function. The main challenge for the modeller is which models to use and how to integrate them together.

Economic scenarios are already widely used with cashflow models. They are used to test how profit variability is influenced by systematic risk. Each economic scenario will result in a different *idealistic* profit.

Economic scenarios can also be used to calculate risk-adjusted prices using deflators or other risk-neutral techniques.

Many industries also invest a great deal of effort in understanding the operational risks that they face. Examples of operational risk models include.

- Oil Exploration Models
- Psychological Models of Fraud Behaviour
- Catastrophe Risk Models

These risks are largely uncorrelated with the economy and therefore qualify as unsystematic risks. Sophisticated stochastic models enable the company to understand how its profit is influenced by operational risks. These models are useful in understanding the relationship between the expected value of frictional costs and the variability of *idealistic* profit.

A comprehensive solution for modelling a company would be to combine economic and operational scenarios into a single model. We are aware of three techniques for combining economic and operational risk scenarios.

1. Economic and operational scenarios can be run orthogonal to each other by running all the operational scenarios for each economic scenario. A frictional cost function can be added to make the cashflow model *realistic*.
2. Each operational scenario can be matched with one economic scenario to create a set of scenarios in which the operational and economic risks vary together. The matching can be done to reflect the correlation between the operational risk and the deflator. A frictional cost function can be added to make the cashflow model *realistic*.
3. Only economic scenarios are used and the severity of the frictional cost function is increased to make the cashflow model *realistic*. In this situation, the frictional cost function is working harder to take account of the frictional costs than when operational risk scenarios are included. This option is attractive if a simpler model is required.

9 MANAGEMENT TECHNIQUES & FRICTIONAL COSTS

9.1 Introduction

Frictional costs can help us understand some of the valuation and profit target techniques that have evolved in the insurance industry and elsewhere.

We start this section looking at techniques involving corporate utility and typically investor utility. We then discuss unsystematic risk premiums and insurance valuation. Finally we consider how project hurdle rates can destroy shareholder value.

9.2 Corporate Utility

Corporations consist of many stakeholders. Utility functions are usually associated with individual preferences of the stakeholders. Some management techniques have developed that suggest corporate strategy should be based on maximizing the utility of a corporation. Can we understand why this might have been done?

Our understanding of frictional costs can help us. Risk averse investors can increase their expected utility by mitigating some risk; for example by taking out personal lines insurance. Similarly, companies can add value to their shareholders by reducing frictional costs using risk mitigation; for example commercial lines insurance. One way of rationalizing this behavior is to reason that companies have utility functions. Consideration of frictional costs is able to rationalise this behavior by only considering investor utility functions.

We can illustrate this by plotting each approach on a schematic showing reward for systematic vs. unsystematic risk.

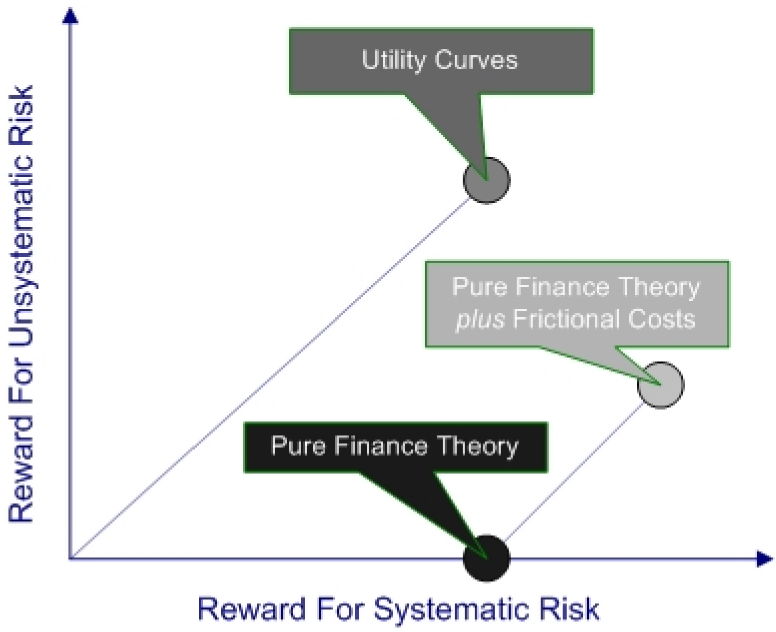

Figure 9-1

We can see that utility functions give extra reward for systematic and unsystematic risk while pure finance theory gives reward only for systematic risk. However, pure finance theory *plus* frictional costs rewards systematic risk and unsystematic risk to the extent that it affects shareholder valuation.

9.3 Typical Investor Utility

Similarly, management techniques have developed which make an assumption about the utility function of a typical investor. These techniques seek management strategies to optimize the utility of the typical investor. However, investors can arrange their assets to choose a desired cashflow pattern. If an investor wanted a less risky share they could invest more in risk-free assets or choose a different stock.

Frictional costs can help us understand the motivation for this approach by using the same example discussed in the previous sub-section. The difference is that company behavior is rationalised by assuming that all shareholders have the same utility function. Consideration of frictional costs is able to rationalise company behavior by considering the utility functions of all risk-averse investors.

9.4 Unsystematic Risk Premiums

Unsystematic risk does not attract a risk premium because it is diversifiable. Yet some valuation techniques calculate unsystematic risk premiums which are added to discount rates to reduce values.

Our understanding of frictional costs can help us again. The convex nature of frictional costs means that a more volatile profit due to unsystematic risk will lead to an increase in the expected value of frictional costs.

Some methods for calculating risk premiums for unsystematic risk may get a numerical answer that is close to the market price. This could happen if the unsystematic risk premium was well correlated with the percentage reduction in value due to frictional costs.

9.5 Insurance Valuation Techniques

In finance theory assets inside a portfolio are worth the same as they are outside a portfolio. Were this not the case an arbitrage opportunity would exist. Insurance valuation techniques calculate the reserves that need to be put aside for a policy. These techniques put a different value on an insurance policy held in a portfolio of similar policies, than one held individually. The argument used is that the liability to the insurer is lowered because of the diversification effect of the portfolio. According to finance theory this would appear to be an arbitrage opportunity.

Our understanding of frictional costs can explain this apparent anomaly. There are significant frictional costs associated with writing insurance policies which do not exist in holding financial assets. The costs of ownership of financial assets are almost zero, whereas writing an insurance policy incurs costs such as regulatory capital, sales costs, claims costs and moral hazard. The cost per policy falls as the number of policies increases.

By implicitly deducting these costs from the price of the underlying insurance risk, insurance valuation techniques makes it appear that the value of an insurance policy lowers when it is held in a portfolio.

The reason this is not an arbitrage is because the insurance contract cannot be sold without taking ownership of the costs associated with the contract. In the absence of frictional costs we would expect insurance contracts to be valued in the same way as financial assets.

9.6 Hurdle Rates

A common incentive technique for businesses is to set hurdle rates that projects must expect to return in order to receive funding. The intention is to provide an incentive to business units to increase the return for shareholders. This type of profit target can achieve the opposite effect and destroy value.

Let us assume that managers can find projects which have an expected return that exceeds the hurdle rate. These projects will, most likely, have a higher expected return because they have more risky income streams. Ignoring frictional costs a project will add value if its expected return exceeds the risk premium required for its level of systematic risk. If the expected return is *lower*, the project will destroy value even without considering frictional costs.

Now consider the frictional costs. The increase in project risk could lead to an increase in profit risk, if the project cashflows are positively correlated with company profit. The convex nature of frictional costs means an increase in profit volatility will increase the value of the frictional costs. If the increase in the value of frictional costs is greater than the added value from the project, there will be a reduction in shareholder value.

As an example consider a project that earns a risk-free return that is 5% above the prevailing risk-free rate. This project is clearly adding value for the company. However if a hurdle rate is set 10% above the risk-free rate, this profitable project would be likely to have its funding diverted to a project that exceeds the hurdle rate. If the cashflows from the riskier project are positively correlated with the company profit the company's frictional costs will increase. Once the frictional costs are taken into account the riskier project could well be destroying shareholder value. The net result of applying the hurdle rate would therefore be to destroy shareholder value.

An optimal value creation strategy could be to distribute capital until the marginal return on each business unit was equal. Frictional cost functions can help in this process by allocating frictional costs based on the marginal contribution to the frictional costs at corporate level.

10 GENERAL INSURANCE APPLICATIONS

by Rolf van den Heever

10.1 Introduction

We have seen that frictional costs aim to quantify all the residual items that have not been included in a valuation model. These residual items include allowance for non-systematic risks.

We have also seen that the risk discount rate can be reengineered to include an additional risk premium to compensate for frictional costs. We know from financial economic theory that unsystematic risk does not earn a reward, rather unsystematic risks lead to additional costs that are often not encapsulated in the valuation modelling.

In this chapter we consider potential dangers that may arise from an incorrect interpretation or omission of frictional costs in the general insurance industry.

10.2 Strategic Decisions

10.2.1 Setting Capital Requirements

Some general insurance companies and reinsurance companies set capital requirements based on the variability of expected future profit streams. As the expected variability increases, a higher level of capital is required to minimise the probability of ruin.

Companies are required to set capital at least equal to regulatory requirements. These capital requirements seem to be converging on a percentile of expected loss or alternatively a level of shareholder shortfall. Such requirements aim to reduce the level of risk inherent in the company's cashflows.

Both managers and regulators are concerned about the bankruptcy costs. Given the high cost of entry, shareholders will require a level of capitalisation to minimise the potential bankruptcy costs. Regulators wish to ensure that all policy commitments can be maintained.

Though a consideration of the probability of ruin is appropriate this can, however, lead to the interpretation of an average investor and this investor's utility function. Managers will interpret what the risk appetite of this typical investor is and therefore the expected return required by this typical investor.

10.2.2 Setting Profit Targets

Profit targets can be set based on the efficient frontier. These targets are set based on the risk-free rate plus the company specific risk premium.

Some companies base their appraisal of the company specific risk premium on the variability of the future profit streams. Therefore the Beta used for the CAPM is based on the variability of expected future profit streams.

This Beta is not appropriate as it will include non-systematic risk as well. Therefore the profit target will be set with an allowance for a reward for unsystematic risk. We have seen that such a reward is not required as shareholders are able to diversify this risk. However, we have also seen that the company will incur frictional costs that are not included in the model.

The profit target should be set based on the systematic risk. We have already mentioned that these profit targets seem too low. That is to say they yield valuations for the company higher than the market price. When no allowance is made for frictional costs in the model, the profit target has to be reengineered to ensure that the valuation equals the market price.

10.2.3 Setting Management Objectives

Some management objectives are set based on the profit target. We know that the expected return of a set of cashflows increases with the increase in risk of the cashflow. We also know that it is possible for shareholders to reverse this increase in risk by selecting a diversified investment portfolio. Neither of these changes result in any increase or decrease in value.

Given that management objectives are set based on profit targets, managers will aim to increase the risk inherent in the business. Examples are available of reinsurance companies changing their business practice to focus predominantly on non-proportional lines of business rather than proportional lines of business. Such actions increase the expected return on investment at the price of the increased risk. Again an investor would be able to diversify through lower investment in the reinsurance company and more in the direct company which now retains the quota share business.

We have seen that it is more likely that increasing the risk inherent in the business will lead to additional frictional costs. This will destroy value.

10.2.4 Regulatory Arbitrage

Regulators require a safety margin in the capital supporting a company's business. From a shareholder's perspective, the same level of diversification could be achieved by investing in risk-free assets and riskier insurance operations.

To the extent that the regulatory requirements result in additional payments made to staff or consultants to ensure compliance, these payments reduce shareholder value. It is understood that this cost is required to protect the interests of policyholders. Shareholders will nevertheless aim to mitigate these frictional costs. By relocating to other areas with less stringent controls shareholders can instantly generate value.

10.2.5 Allocating the Cost of Capital

Many companies allocate the cost of capital for measurement and incentive purposes. Allocating a risk premium for unsystematic risk is often questioned by managers, not because it arises from unsystematic risk but because the cost seems unrealistically high. We feel that the allocation of a frictional cost is intuitively more apparent.

The process of allocation of frictional costs is the same as the process of allocating the cost of capital. That is to say the marginal contribution of the cost. We represent this as the correlation of a business line specific cost to the company specific cost.

10.3 Valuation

Setting a yield requirement based on the variability of expected profit streams is only appropriate if no other investments were available to shareholders. In such an instance the variability inherent in the business would be akin to the variability inherent in the market. Given that shareholders will be able to diversify, such an approach is not appropriate.

Practitioners will often find that the valuation of the company derived through the present value of expected future profits, does not seem consistent with market valuation. We have noted that the reason for such discrepancies arise from:

a) non-linear tax costs

b) agency costs

c) bankruptcy costs

We termed these items frictional costs. We also saw that discrepancies could arise from unfounded management optimism and model approximations.

We have also noted that in the past these frictional costs were allowed for by incorporating a company specific risk premium. Such a risk premium is not justifiable from a shareholders point of view but rather from a model deficiency point of view. Fortunately, the same result is achieved.

By considering the factors that might give rise to frictional costs, it will be easier to allow for frictional costs in the valuation methodology. When considering the risk premium approach it has to be recognised that the frictional costs explicitly represent model error whereas the risk premium does not. That is to say the risk premium will reduce as the model improves. This does not make intuitive sense.

The recent regulatory developments to quantify operational risks will therefore aid the modelling and reduce the frictional cost component even further.

10.4 Frictional Costs and Non-Systematic Risk

How do we allow for the risk loadings in reinsurance pricing for example proportional hazard loadings or standard deviation loadings? We do not intend to discuss the merits of these approaches here but wish to consider them in the context of frictional costs.

If we consider reinsurance excess of loss products, we can see that as the non-systematic risk increases so the frictional costs increase. For example writing a catastrophe cover gives rise to potential bankruptcy costs and the higher the degree of risk inherent in the catastrophe cover, the higher the potential bankruptcy costs and correlation to company wide frictional costs.

For the purpose of this paper we do not establish the allocation of frictional costs to individual products but can note that there is a correlation between non-systematic risk inherent in a product and the correlation of that product's frictional cost to the company's frictional costs.

10.5 Summary

We have seen that some of the management strategies and valuation methodologies currently applied in practice can be explained according to financial economic theory in conjunction with our understanding of frictional costs.

We have also seen that some strategic decisions appear to be flawed given the ideas developed in this paper.

11 PETROLEUM INDUSTRY APPLICATIONS

by Olivier Allen

11.1 The Petroleum Industry Challenge

Oil and gas company management face increasing competition to attract capital and create shareholder value. High oil prices and the pressure to achieve financial targets are shifting focus from volume growth to value based strategies. Improved valuations and subsequently better capital allocation is at the heart of value creation. In this example we show how management can use deflator models together with a frictional cost function to maximise the market value of their company.

11.2 Capital Allocation Optimisation

In order to illustrate the importance of frictional costs, we look at a portfolio of twelve North Sea oil producing assets. The portfolio has a market value of $1.8 billion and the company uses a hurdle rate of 7% as an investment decision policy.

If we calculate the consolidated NPV using a standard discounted cashflow analysis we obtain a portfolio value of $2.4 billion (Figure 11-1 illustrates the value of individual assets). This is above the market value. Using a deflator model, we can perform a risk-adjusted valuation according to the level of systematic risk at asset level. We obtain a value of $2.28 billion. Although the deflator model is allowing for the level of systematic risk correctly we are still overvaluing the portfolio.

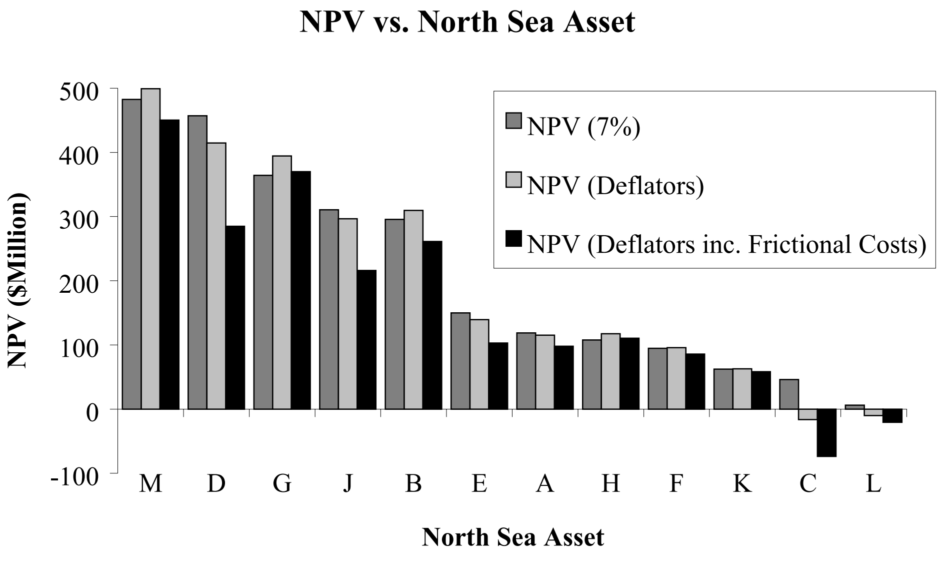

Figure 11-1 - North Sea oil and gas assets valuations using three different methods

To reconcile the market value with the cashflow projections, we must allow for frictional costs. These are cashflow deductions from profits that financial markets take into account in valuation. We modelled the difference between the deflator valuation and the market value using a simple profit dependent frictional cost function. This allowed us to distribute the frictional costs back to every asset in the portfolio in coherent way that recognised the contribution of each asset to the frictional costs at company level. Portfolio assets with higher contributions to future profit variability receive a higher proportion of the frictional costs.

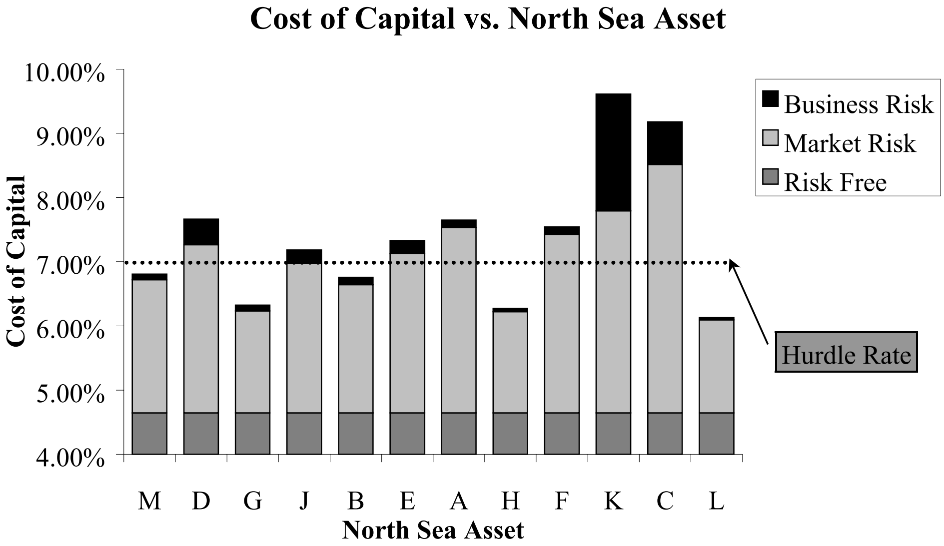

Figure 11-2 - Impact on the cost of capital of adjusting for systematic risk and frictional costs

Figure 11-2 illustrates the reverse-engineered cost of capital that would have been required for each asset in order to reconcile the market valuation with the cashflow model. The cost of capital is split between the risk-free rate, the return for systematic risk on the unadjusted model, and the extra return for frictional costs.

Asset D represents a fifteen year project. It is the most capital intensive and largest project in the portfolio. Due to its large contribution to the portfolio profit and risk, this project receives 27% of the total present value of the frictional costs. Although this has a small impact on its cost of capital (Figure 11-2), it reduces the net present value of this asset by $130million (Figure 11-1). The inclusion of the frictional costs also makes Asset D also a less profitable investment than originally thought.

A management whose objective is to maximise shareholder value could use a deflator model together with a frictional cost function to optimise capital allocation and maximise shareholder value.

11.3 Hedging Strategies

Oil and gas companies may enter into forward oil sales contracts to protect their earnings against a fall in oil prices. Hedging strategies can contribute to value creation or degradation depending on the balance between the costs of hedging and the associated reduction of the costs of risk. In order to add value, the cost of hedging (broker fee, spread cost and management time) needs to be balanced against a reduction in frictional costs.

The frictional cost benefit of hedging includes the reduction of expected tax liabilities, bankruptcy and financial distress costs and the reduction of funding costs by providing internally generated cash when the company needs it.

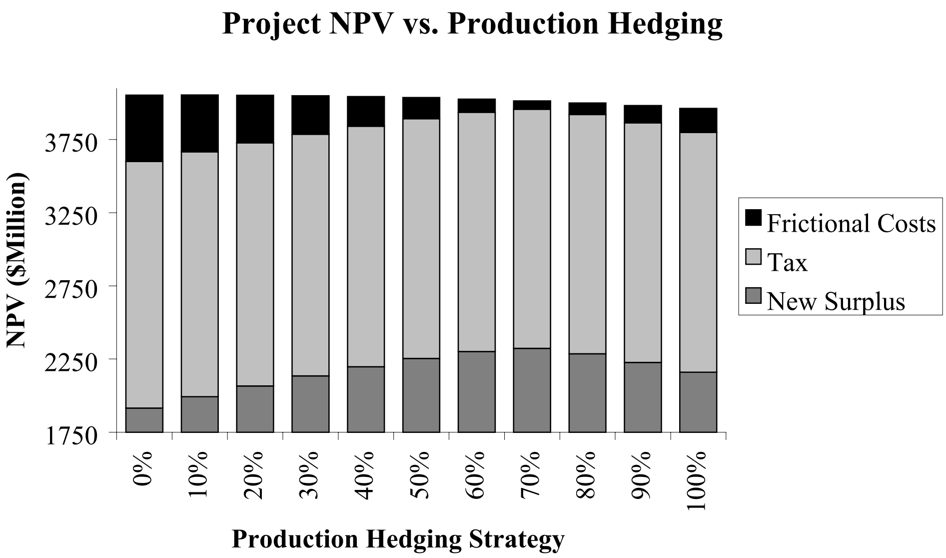

Figure 11-3 - Profitability index difference between two valuation methods

Using our portfolio of 12 North Sea oil producing assets, we investigated different hedging strategies where the company could sell different percentages of its production forward. Figure 11-3 and Figure 11-4 illustrate the results. In this example, the stakeholders are the tax authority, the shareholders and a third group who benefit from the frictional costs. Figure 11-3 shows that hedging is creating value for shareholders at the expense of the other stakeholders.

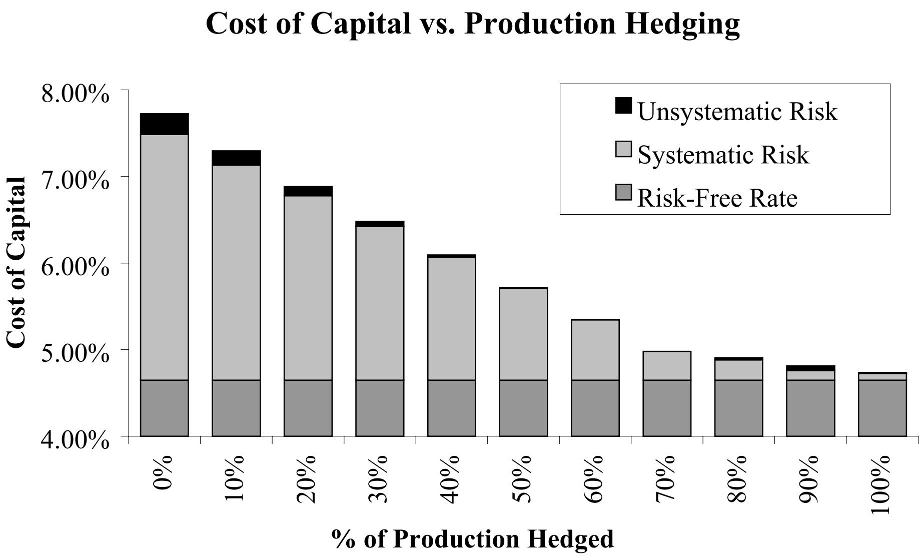

Figure 11-4 - Impact of different hedging strategies on the portfolio cost of capital

Extending our analysis, it is possible to find the optimum hedging strategy that will maximise the company's market value. In this example, selling 70% of the production forward will maximise the net surplus available to shareholders.

The modelling of frictional costs in the context of the oil and gas industry can help management optimise the value of their portfolio. Some of the practical benefits involve the optimisation of capital allocation and hedging strategies.

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13 APPENDIX A - UTILITY FUNCTIONS

Economics recognises two characteristics of typical investors:

- Non-Satiation: They prefer more wealth to less.
- Risk Aversion: They derive less satisfaction per unit of wealth the more wealth they have.

The standard approach to capturing these characteristics is the utility function. The utility function requires the following features to capture the characteristics above:

- Monotonically Increasing (Non-Satiation)
- Concave (Risk Aversion)

A commonly used utility function uses the natural logarithm, illustrated below.

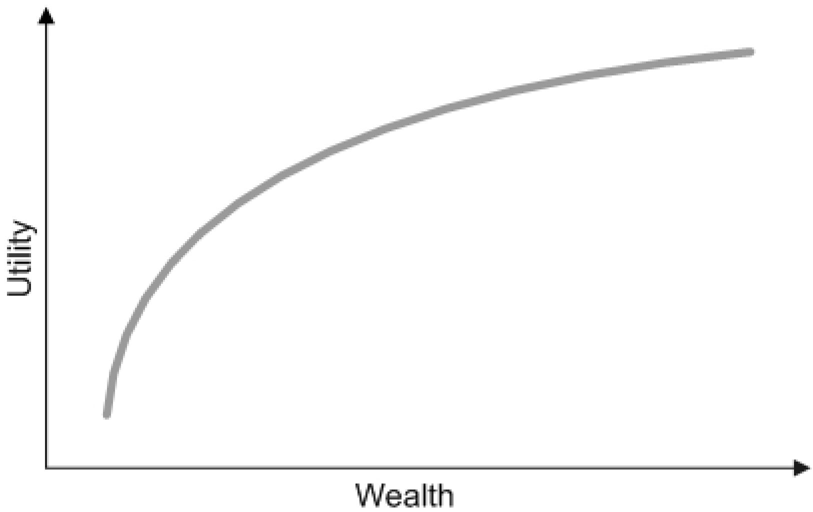

Figure 13-1

Economic theory says that an investor will be indifferent between two assets if they provide the same expected utility. It also expects investors to maximise their expected utility.

Consider a two-state, one time period model. Each state is equally probable. It has two assets. Each has an expected payment of 1. One asset is risk-free and guarantees payment of 1. The other asset pays 1.50 in one state and 0.50 in the other state. We summarise this below:

	State 1 Cashflow	*State 2 Cashflow*	*Expected Cashflow*
Risky Asset	1.5	0.5	0.5(1.5+0.5) = 1
Risk-Free Asset	1.0	1.0	1

Table 13-1

Consider an investor with a log utility function. We can calculate the utility derived for each asset in each state. By taking averages we can also calculate the expected utility derived for each asset. We summarise this below.

	State 1 Utility	***State 2 Utility***	***Expected Utility***
Risky Asset	Log(1.5) = 0.405	Log(0.5) = -0.693	0.5(0.41-0.69) = -0.140
Risk-Free Asset	0.00	0.00	0.00

Table 13-2

We see that the expected utility of the risk-free asset exceeds the expected utility for the risky asset. In this scenario the investor would always choose the risk-free asset. The investor could be persuaded to buy the risky asset if the expected utility of the risky cashflows was increased to match the expected utility of the risk-free asset. Because the utility function is upward sloping this is only possible by increasing the risky cashflows. This is equivalent to adding a positive risk premium.

In conclusion, economics is telling us that a risk-averse investor will require a positive risk premium as compensation for a risky cashflow. The risk premium required increases with the risk of the cashflow.

14 APPENDIX B - SECURITISATION

Securitisation is the splitting of ownership of an asset into a large number of tradable shares. Corporate dividends and debt are typically split into securities and traded in liquid markets. Insurance risks are typically not split into securities although some element of securitisation occurs for very large risks in the London Market.

Securitisation makes it possible to replicate, on a small scale, the returns experienced from holding all the risks in the market. By replicating the returns of the entire market investors will only be subject to systematic risk. To a lesser extent reinsurance contracts serve a similar purpose to securitisation in insurance markets.

Securitisation occurs where the diversification benefits of providing a market outweigh the costs of sharing ownership. The large market in world shares and the relatively low cost of issuing shares means that share markets do well. Other markets have had more problems though. Attempts at securitising insurance risks have had mixed fortunes. Often the costs of dealing with issues such as moral hazard outweigh the diversification benefits of what is initially a small market.

15 APPENDIX C - SYSTEMATIC RISK

15.1 Introduction

In this section we will show how the risk premium on an asset is related to the covariance of the asset return with the state price deflator. We will then work through a simple example using the Feast & Famine model.

15.2 Algebraic Proof

We start with the deflator pricing equation as follows, where P_X is the risk-adjusted price of a risky cashflow, X.

$$P_X = E[DX]$$

Equation 15-1

We can rewrite the expression above as follows.

$$E\left[D\frac{X}{P_X}\right] = 1$$

Equation 15-2

Deducting *E[D]* from both sides, we get the following expression.

$$E\left[D\frac{X}{P_X}\right] - E[D] = 1 - E[D]$$

Equation 15-3

Combining the two previous expressions we can write the following.

$$E\left[D\left(\frac{X}{P_X} - 1\right)\right] = 1 - E[D]$$

Equation 15-4

We know that the return is described by the following expression.

$$R_X = \frac{X}{P_X} - 1$$

Equation 15-5

Combining the two previous expressions we can write the following.

$$E[DR_X] = 1 - E[D]$$

Equation 15-6

The definition of covariance between the return and the deflator is given by the following expression.

$$COV[R_X, D] = E[R_X D] - E[R_X]E[D]$$

Equation 15-7

Combining the two previous expressions we can write the following.

$$COV[R_X, D] + E[R_X]E[D] = 1 - E[D]$$

Equation 15-8

Dividing throughout by *E[D]* we can write the following.

$$\frac{COV[D, R_X]}{E[D]} = \frac{1}{E[D]} - E[R_X] - 1$$

Equation 15-9

We recall the definition of the expectation of the deflator is as follows, where *r* is the risk-free rate.

$$E[D] = \frac{1}{1+r}$$

Equation 15-10

Combining the two previous expressions we can see that the expected return is given by the following expression.

$$E[R_X] = r - \frac{COV[D, R_X]}{E[D]}$$

Equation 15-11

Deducting the risk-free rate from both sides gives us the following expression for the risk premium.

$$E[R_X] - r = -\frac{COV[D, R_X]}{E[D]}$$

Equation 15-12

15.3 Feast & Famine Example

We now demonstrate the relationship between risk premiums and covariance of the return with the deflator. We use the Feast & Famine model.

We use the deflator and the returns calculated in Section 2 to calculate the covariance of the share return with the deflator.

X	***COV(R_X,D)***
A	0.5(0.7×0.818+1.2×-0.394)-0.95×0.212 = -0.152
B	-0.188

Table 15-1

We summarise the risk premiums and the covariance calculations in the following table.

X	***Risk Premium***	***-COV(R_X,D)/E[D]***
A	0.159	0.152/0.95 = 0.159
B	0.197	0.188/0.95 = 0.197

Table 15-2

We see that the risk premium is precisely the negative of the covariance of the return with the deflator, divided by the risk-free discount factor. This is a standard result in financial economics.

The deflator is proportional to the marginal utility of a risk-averse investor with an optimal portfolio. This means that a risk premium will be positive for an asset that tends to pay out more when marginal utility is low (i.e. in high states of wealth).

16 APPENDIX D - MODIGLIANI & MILLER

16.1 Introduction

In this appendix we work through a simple Modigliani & Miller example using the Feast & Famine model. It is intended as a primer for people unfamiliar with the Modigliani & Miller result.

16.2 Example

Consider Company A from the Feast & Famine example. Let us assume that Company A is currently 100% financed by equity, such that the income is all distributed to shareholders.

We now consider what would happen if the company decided to move to 50% equity funding and 50% bond funding. Company A would buy up 50% of its stock for 0.825, the prevailing market price, and fund the purchase using 0.825 raised from a bond issue.

The respective share and bond investments now have the following prices and payments.

	Feast Payment	***Famine Payment***	***Price***
Share	2.132	0.132	0.825
Bond	0.868	0.868	0.825

Table 16-1

We have constructed the bond payments in this table by considering the risk-free return required on the company's bond. The yield is risk-free because in this example there is no risk of default. We constructed the equity payments by deducting the bond payments from the Company A income.

Consider an investor who was happy with the original payment pattern from Company A shares. The payment pattern for the share income is now more risky because of the introduction of the bonds. We might be tempted to measure the risk by calculating the variance of the return. However, the correct measure of risk is the covariance of the return with the deflator. Therefore this is what we need to calculate to demonstrate that the risk has increased.

First we calculate the returns for Company A with and without the debt.

R(X)	***Feast***	***Famine***	***E[R(X)]***
Without Debt	$3/1.65-1 = 0.818$	-0.394	$0.5(3+1)/1.65-1 = 0.212$
With Debt	1.58	-0.840	0.372

Table 16-2

Next we calculate the negative of the covariance of the return of the deflator with Company A, with and without debt.

R(X)	*-COV(*R_X, D*)*
Without Debt	0.5(0.7x0.818+1.2x0.394)-0.95x0.212 = 0.152
With Debt	0.5(0.7x1.58+1.2x-0.840)-0.95x0.372 = 0.303

Table 16-3

Table 16-3 shows us that the risk for the shareholder has increased substantially by adding the debt.

The investor can however recover his previous position simply by forming a capitalisation-weighted portfolio of the debt and equity issued by the company. This is shown in the following table.

	Feast Payment	***Famine Payment***	***Price***
Equity	2.13	0.132	0.825
Debt	0.868	0.868	0.825
Equity + Debt	2.13+0.868 = 3	0.132+0.868 = 1	0.825+0.825=1.65

Table 16-4

This restores the investor's cashflows to the pre-debt position for the original cost of the share. In this way investors can effectively undo the financing decisions of companies they are invested in. It implies that the company does not need to take account of the investors preferred level of leverage when making its financing decision.

The cost of financing a company is often measured as the Weighted Average Cost of Capital (WACC). The WACC is a weighted average of the return required by shareholders and the return required by bondholders. This is expressed in the formula below where, C_{SHARE} is the share capital, B_{SHARE} is the bond capital, R_{SHARE} is the return required by shareholders and R_{BOND} is the return required by bondholders.

$$WACC = \frac{C_{SHARE}R_{SHARE} + C_{BOND}R_{BOND}}{C_{SHARE} + C_{BOND}}$$

Equation 16-1

Notice that the valuation using deflators has maintained exactly the same valuation for the company despite substituting equity finance for cheaper debt finance. We can see why if we calculate the expected return for the shareholders as shown in the table below.

	100% Equity	***50% Equity / 50% Bonds***
Shareholders Expected Return	0.212	0.372
Bondholders Expected Return	N/A	0.0526
WACC	0.212	$(0.825\times0.372+0.825\times0.0526)/(0.825+0.825) = 0.212$

Table 16-5

We see that the WACC remains unchanged by the replacement of equity with debt. The rise in expected return demanded by shareholders exactly cancels out the lower return demanded by bondholders.

The result above is the main conclusion of the 1958 paper by Modigliani & Miller. They concluded that the financing decision of real companies does not affect the WACC.

17 APPENDIX E - FRICTIONAL COST FUNCTION FAMILIES

17.1 Introduction

In this section we set out the necessary conditions for a family of frictional cost functions to work as described in this paper.

17.2 Frictional Cost Families

A family of frictional cost functions, Θ, contains all possible frictional cost functions, $\theta(x)$, that could be chosen using a particular frictional cost formula. In the case of the simple frictional cost function used in this paper, a family would include all choices of the α and β parameters for a particular value of K.

A frictional cost function family, Θ, will be suitable if all the functions, $\theta(x)$, in the family satisfy the following conditions for constant $c > 0$.

$$\theta(x) \in \Theta$$

Equation 17-1

A general condition to ensure that translation invariance axiom will hold is given below. The axiom will hold exactly when the minimised frictional cost function is used.

$$\theta(x + c) \in \Theta$$

Equation 17-2

A general condition to ensure that positive homogeneity axiom will hold is given below. The axiom will hold exactly when the minimised frictional cost function is used.

$$\frac{1}{c}\theta(cx) \in \Theta$$

Equation 17-3

Recall that in the paper we chose the α and β parameters so that they would minimise the risk measure $E[D\theta(x)]$ for a given value of K. This we ensure that the risk measure using of any other frictional cost function from the same family will be bigger than or equal to the risk measure using the minimised frictional cost function.

17.3 Demonstrating the Marginal Allocation

Since $\theta(x)$ is chosen such that $E[D\theta(X)]$ is minimised, we can write the following expression which is a strict equality when $c = 1$.

$$\frac{1}{c}E\left[D\theta\left(cx\right)\right]\geq E\left[D\theta\left(x\right)\right]$$

Equation 17-4

Now we try to minimise $E[D\theta(x)]$ by differentiating it with respect to c and equating it to zero.

$$-\frac{1}{c^2}E\left[D\theta\left(cx\right)\right]+\frac{1}{c}E\left[Dx\theta'\left(cx\right)\right]=0$$

Equation 17-5

We know from Equation 17-4 that when $E[D\theta(x)]$ is minimised $c = 1$. We can substitute this into the expression above and rearrange to give the following relationship.

$$E\left[D\theta\left(x\right)\right]=E\left[Dx\theta'\left(x\right)\right]$$

Equation 17-6

17.4 Example

17.4.1 Introduction

In this section we illustrate why the marginal frictional cost allocation works and other techniques do not work.

Let's assume that we have a company, M, that is made up of two separate business units, A and B. The *idealistic* profit for Company M is the sum of the profits of Business Unit A and Business Unit B. If we let X_M denote the *idealistic* profit of Company M, and X_A and X_B denote the *idealistic* profits of the business units, we can write the following.

$$X_M = X_A + X_B$$

Equation 17-7

We now need to allocate the expected value of frictional costs to Business Unit A and Business Unit B such that their sum adds up to the frictional costs for Company M. We can express this as follows, where F is used to indicate the expected value of frictional costs.

$$F_M = F_A + F_B$$

Equation 17-8

We now consider three approaches to allocating the frictional costs to each of the business units.

17.4.2 First Attempt

We try allocating the stand-alone frictional cost to each line. This can be expressed as follows.

$$F_A = E[D\theta(X_A)] \ : \ F_B = E[D\theta(X_B)]$$

Equation 17-9

Adding these two expected frictional costs, we find that the total is bigger than or equal to the frictional costs of Company M. This is because $E[D\theta(X)]$ satisfies the inequality subadditivity risk measure axiom. We can express this as follows.

$$F_A + F_B = E[D\theta(X_A)] + E[D\theta(X_B)] \geq E[D\theta(X_M)]$$

Equation 17-10

17.4.3 Second Attempt

We now try allocating frictional costs using the frictional costs of Company A less the stand-alone frictional costs of the *other* business unit. This can be expressed as follows.

$$F_A = E[D\theta(X_M)] - E[D\theta(X_B)] \ : \ F_B = E[D\theta(X_M)] - E[D\theta(X_A)]$$

Equation 17-11

Now we find that the sum of the allocated frictional costs are less than or equal to the frictional costs of Company M. This is again because $E[D\theta(X)]$ satisfies the inequality subadditivity risk measure axiom.

$$F_A + F_B = 2E[D\theta(X_M)] - E[D\theta(X_B)] - E[D\theta(X_A)] \leq E[D\theta(X_M)]$$

Equation 17-12

17.4.4 Third Attempt

Finally we try allocating the frictional costs using the marginal risk measure. This can be expressed as follows.

$$F_A = E[DX_A\theta'(X_M)] \ : \ F_B = E[DX_B\theta'(X_M)]$$

Equation 17-13

Adding the allocated frictional costs we get the following expression.

$$F_A + F_B = E[DX_A\theta'(X_M)] + E[DX_B\theta'(X_M)] = E[D(X_A + X_B)\theta'(X_M)] = E[DX_M\theta'(X_M)]$$

Equation 17-14

We know from the previous sub-section that the following expression holds if we choose parameters to minimise the expected value of the frictional costs.

$$E[D\theta(X_M)] = E[DX_M\theta'(X_M)]$$

Equation 17-15

Using the expression above we can write the following.

$$F_A + F_B = E[D\theta(X_M)]$$

Equation 17-16

Therefore the marginal risk measure is the correct allocation method.