Fair Value Accounting: Implications for General Insurers

By Julian Leigh

Acknowledgements

This paper was written to develop principles set out in a paper presented to the 2003 GIRO convention and to examine their practical implications. I would like to thank the authors of that paper for their work; without it this paper would never have been written.

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I would like to thank several of my colleagues, who have reviewed this paper. All of them made helpful comments and suggestions, which have improved the paper.

I am grateful for the support of my employer in providing the time and facilities needed to complete this paper.

Notwithstanding the above, I take full responsibility for the ideas and results set out in this paper. The opinions and results set out in this paper should not be regarded as the views of my employer or of any other person.

Abstract

This paper describes briefly the background to fair-value accounting in insurance. It discusses the principles set out in a paper presented to the GIRO Convention 2003 and develops them into a formula for the calculation of fair-value reserves. The implications of the formula are then considered, in terms of the amounts of reserves that are likely to be required, the effects on the emergence of profit and the likely volatility in reserve amounts and declared profit in situations where assets and liabilities are unmatched and those where they are well matched.

1. Introduction

The International Accounting Standards Board (IASB) has been preparing a new International Accounting Standard for insurance contracts for some years. This has now reached the stage of an Exposure Draft[i] (ED5). However, the IASB's project has been divided into two phases, and the most important changes will emerge in Phase II of the project; ED5 is a draft of the standards to be introduced in Phase I. Chief among the changes contemplated in Phase II is the use of fair values for the assets and liabilities, principally technical provisions, arising from insurance contracts. It was originally proposed that fair values would have to be disclosed in accounts as part of Phase I, and therefore would need to be calculated as at 31 December 2006, but this proposal has been dropped in the light of comments received on ED5.

It is not certain that fair values will be used as the basis of accounting, even in Phase II. Senior figures in the insurance industry have objected strongly to the probable increase in volatility of results that will arise from their use. However, the principle of fair values appears to be well entrenched in IASB thinking, and it will take a major change in approach if a different concept is to be adopted.

Fair values may be either "entry values" or "exit values". Entry values are derived from the price at which an insurer sells its product: the premium. It appears to be probable that accounting will be on the basis of entry fair values until a policy has expired. After that they will be based on the exit fair values, and it is in this way that the term "fair value" will be used in this paper, which is concerned with the fair value of the liability to pay a tranche of general insurance claims that have already occurred.

Fair values are defined as market values. The full definition is "the amount for which an asset could be exchanged, or a liability settled, between knowledgeable, willing parties in an arms' length transaction"[ii]. For most of the main asset classes used by general insurers this presents little difficulty: prices of equity shares, gilts and bonds are quoted, or, if particular securities are not, can generally be valued on a market-value basis by reference to those that are. However, general insurance liabilities are generally not exchanged in a liquid market, certainly not one that gives public quotations. For this reason we have the concept of market-consistent valuation, the setting of provisions for outstanding claims and other technical liabilities at amounts that would be consistent with this criterion if such markets did exist.

It is evident that a rational purchaser[1] would not accept an amount that was less than needed to pay off the liabilities he was assuming. Indeed, because he is accepting an uncertain, and therefore risky, liability, he would normally be expected to seek more than this, while the seller, since he would be giving up a risky liability, may be prepared to pay more. Financial economic theory suggests that in a liquid market this premium charge for uncertainty would be bid down to nil, to the extent that risks are diversifiable by a shareholder in the company concerned. However, in this paper I have assumed an interpretation of fair value that accepts that a deep, liquid market does not exist, and this seems to be the interpretation required by the IASB.

The amount needed to pay off the liabilities would be the discounted value of the best estimate[2] of the liabilities. The rate of discount used should be a risk-free rate appropriate to the term and currency of the liability; if the purchaser achieves a higher rate on investing the fair value it is properly a return to his investment strategy, not to the insurance transaction. The difference between the fair value (the notional purchase price) and the discounted best estimate is known as the market-value margin.

The calculation of fair value and the market value margin has been much discussed in recent years. In a paper to the 2003 GIRO convention, White et al[iii] set out what they described as a practical suggestion for the implementation of fair values. This paper examines their proposals, develops their principles into a formula for fair values, and investigates some of the implications for a company writing long-tailed motor bodily injury liability business should they be adopted.

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[1] In this paper the term "purchaser" will be used to denote the person or institution acquiring the liabilities, even though he is the recipient of the purchase price, not the payer. "Seller" will be used for the original insurer, who is parting with the liability and paying the fair value.

[2] "Best estimate" is used to denote the arithmetic mean of the distribution of amounts needed to pay the transferred liabilities.

It should be noted that there are several significant, and worthwhile, things that this paper is not about. It makes no attempt to examine whether or not the concept of fair values is a sensible one, but assumes that we will need to make an honest attempt to implement fair-value principles. Also, it is not a paper about reserving. We have assumed in this paper that the payments from a particular set of liabilities will be a sample from a probability distribution with a known mean and with sufficient known about the variability of the outcomes to be able to estimate an appropriate reward for risk. Finally, it is not about whether the fair value should take account of the insurer's own credit standing: it assumes that all liabilities must be paid in full as they arise. These are all worthwhile topics, but they are outside the self-imposed scope of this paper.

2. The GIRO Paper Principles

White et al sets out a suggested method for deriving fair values. This is in four steps.

- Insurers develop mean best estimate cash flow projections for their liabilities.
- A basis is developed for allocating capital to lines of business.
- The rate of return expected on a purchaser's capital should be determined.
- The fair value is the amount which, when added to the capital required, gives the purchaser the required rate of return.

There are some objections to this approach, which are identified in the paper itself. It requires an authority to determine an appropriate amount of capital to be held. The paper suggests that such an authority could be the IASB itself or a national accounting body, and suggest as an example that in the United Kingdom it might be appropriate to adopt the capital requirement factors set out in CP190[iv] (or, presumably, whatever factors are finally adopted), or a multiple thereof.

The adoption of standard amounts of capital by national accounting or supervisory bodies would seem to be unsatisfactory, as one of the main objectives of having standards is transparency between countries, as well as within them. There seems to be little point in having *international* accounting standards if they embody different capital standards in different countries. On the other hand, markets do differ, and a consideration of local conditions may well justify differences between countries. Also, individual classes of business vary substantially within themselves: consider household contents in the United Kingdom and high-layer property damage policies in a hurricane-prone area, both within the property class, for example. This suggests that the capital level should be adjusted to take account of the risk characteristics of the individual book of business being valued. However, doing this would require company-by-company capital standards, and it would be very difficult to ensure that a consistency of approach between companies was achieved. It may be that a finer definition of class is needed to implement this proposal sensibly than is currently available within the FSA returns; the proposed changes to the returns[v] and the introduction of standardised risk categories may help here.

The risk-adjusted rate of return to be achieved by the purchaser suffers from many of the same difficulties. The riskiness of the deal concerned should determine the rate. However, while the market determines a premium for risk in any market where risky commodities are traded, assessing what the level of risk is and what the associated expected extra return may be is not usually possible. Even if they could be observed, relating them to the level of risk in an insurance portfolio is not straightforward.

These objections should not be regarded as being overwhelming. Rough justice may well be better than no justice, and a system that classified business into one of 24 classes for the purposes of capital assessment, as the CP190 proposals do, may well be considered to be sufficiently flexible for practical use. Nor is it impossible to imagine a finer class definition being introduced. There are 71 distinct risk categories set out in annex 5 of CP 202, for example. It should also be borne in mind that these proposed classifications are for companies supervised in the United Kingdom only; international standards would require an agreed international classification, which might have to take account of types of business that were peculiar to particular markets.

The principles do seem to be consistent with the approach to pricing urged in elementary financial economics. The risk that the purchaser accepts by buying the book of business should be adequately compensated by an appropriate risk-adjusted rate of return. The more risky the book, the higher the risk-adjusted rate of return should be. The purchase of a book of business can be regarded as a capital project. The purchaser is required to set aside an initial amount of capital that is appropriate to the book of business. As the liabilities unroll in the future he receives this back, with interest at a risk-free rate, and also receives the market-value margin as it is released, with any investment income it has generated at the risk-free rate, provided that the best estimate turns out to be correct in practice. The discounted best estimate itself and the interest thereon are needed to pay the claims as they fall due, and are not released to the benefit of the purchaser. Any investment profit above the risk-free rate arises from the investment policy of the purchaser, and should not not be considered as part of the insurance transaction.[3]

This logic fails if there is, in fact, no need to commit capital to the project. Such a notion is not ridiculous: the existence of the market-value margin means that the reserve is more than sufficient to pay the claims, at least if the best estimate turns out to be correct. Is it certain that any capital will be required on top of it? Again, recourse to financial economics principles is helpful in answering the question. Suppose that the market-value margin were in itself sufficient to provide a buffer against claims being higher than expected, then the transaction will provide an infinite return to the purchaser, as he receives the market-value margin and interest as profit with no financial outgo, at least on an expected-value basis. In a genuine market, the price of such a bargain would be bid down until the return was appropriate to the level of risk.

Even if there was considered to be no risk at all in the transaction, it would have to yield a risk-free rate of return. In this case fair value equal to the discounted reserves (to call these "best-estimate reserves" might be misleading, as there being no risk implies that the amounts to be paid at each time are known with certainty) will yield the risk-free rate to the purchaser whatever level of capital he subscribes.

A major objection that might be made to this approach is that by relying solely on the expected value of the liabilities it ignores the potential variability as losses run off. It is for this reason that alternative suggestions have been made that involve the stochastic modelling of the runoff of claims, or perhaps work out a market-value margin based on the variance, semi-variance or coefficient of variation of the claims, to be added to the discounted best estimate to get the fair value. However, if these methods produced a lower answer than that produced by following the principles in White et al then they would provide a lower return to the purchaser than the risk level of the transaction required. Consequently, in a market we would expect the price to be bid up until the appropriate return was achieved. The converse would apply if the

[3] We assume that the term "risk-free rate of interest" is well-defined and that the purchaser can actually invest at this rate for the terms required.

methods produced a higher answer. If it is not obvious how the principles allow for the variability of outcomes, it should be realised that a riskier book of business will require a higher capital commitment than a less risky one, and the purchaser will require a higher risk-adjusted return.

It seems reasonable to conclude that these principles make a reliable base for further work.

3. Calculating Fair Value

The principles require that we match the present value of income and outgo for the purchaser of the liabilities. The outgo required of the purchaser is the commitment of the necessary capital, a commitment that must be made at the date of the transaction. The income the purchaser receives is the release of the capital and the market-value margin, and interest generated thereon at a risk-free interest rate, as the claims are paid. The release of the discounted value of the claims payments, the interest thereon and the claims payments themselves may be ignored as they cancel each other out exactly. The present value of the income and the outgo must be equal when calculated at an appropriate risk-adjusted discount rate.

The equation will be critically affected by the speed at which these are released. We might, perhaps, release total assets in proportion to the undiscounted value of future liabilities, the fair value itself or the discounted value of future liabilities. These are the propositions that we will investigate further. Each of them implies a different approach to capital.

However, these approaches are far from unchallengeable. It may well be that the riskiness of the claims increases as time goes by, at least in relation to the amount of the remaining liability, which would justify a slower-than-proportionate release of capital and different parameters in the calculation of the fair value, notably the allowance for risk in the return to the purchaser. Claims that take a long time to settle often have different fundamental characteristics from those in the same class of business that settle quickly, and they will come to dominate the tranche of liabilities as time goes by. Algorithms could certainly be developed that could capture such a procedure, perhaps working recursively from the capital and reserve required to be held at the start of the last period in which claims were due to be paid. These would not yield a simple formula for the fair value and would depend on the particular way in which capital could be withdrawn in any case. (Neither of these is a valid objection to employing such algorithms if the proportionate withdrawal of capital is not thought to be appropriate.)

At the start of the transaction, the purchaser receives the purchase price, which is equal to the fair value (FV). In addition, he must set aside a certain amount of capital

K, to support the runoff. Let us assume that the claims to be paid in each year are C_t (t=1,2, …). Assume for simplicity that these are all payable at the end of each future year.

In the following sections we will examine the implications of the second of the three approaches mentioned above, that the amount of capital held, and therefore the total assets held, be released in proportion as the fair value reduces. The reason for choosing this one is that it is slightly more logically consistent than the others. By setting the initial level of capital as a proportion of the fair-value liability and allowing total assets to fall in proportion with the fair-value liability, the capital held (the total assets held minus the fair-value liability) remains correct according to the initial requirement throughout the runoff. This is not true of the other formulations. However, the difference between the various results is not very large, and the formulae for the other two approaches are developed in the Appendix.

At the end of the year, the total assets held are reduced in proportion as the fair value has reduced. This means that the capital is also held constant as a proportion, k, of the fair value, and we can write K = k FV.

Consider first the case of the fair value a year before the last claim payment. In the original formulation of the problem, it was assumed that **C** was an infinite sequence; in practice we would expect there to be a value of t that was the maximum for which C_t was non-zero. Call this value of t $\underline{t}$, so that $t>\underline{t} \Rightarrow C_t = 0$. Define FV_t as the fair value of the liability at the start of year t. Note that $FV = FV_1$. The capital committed at the start of the last year is k $FV_{\underline{t}}$ and the total assets at the start of the year are (1+k) $FV_{\underline{t}}$. These will generate interest at the risk-free rate, and from the total assets so generated the claims in year t, C_t, must be paid. The assets remaining after the payment of claims revert to the purchaser and must compensate him, with an appropriate rate of return, for committing the capital. This gives the following equation of value.

$$k\,FV_{\underline{t}} = [FV_{\underline{t}}\,(1 + k)\,(1 + i) - C_{\underline{t}}] \,/\, (1 + j)$$

where: i is the risk-free rate of interest

j is the risk-appropriate rate of return on the transaction

Simplifying gives $FV_t = C_t / [1 + i – k (j – i)]$.

Consider now the start of the previous year. There is a similar equation of value, except that it must provide for the payment of FV_t at the end of the year, as well as the year's claims. (Note that the requirement at the end of the year is FV_t, not $(1 + k) FV_t$, as might be thought. At the end of the year the purchaser may exit from the transaction by the payment of FV_t and close the transaction. If he does not do so then he must put up the capital $k\ FV_t$ but by holding the assets for a further year he will gain an appropriate further profit on this capital. This could be considered as a separate transaction.) Otherwise the elements of the equation are the same, but have the subscript t-1 rather than t. This gives the following equation of value.

$$k\ FV_{t-1} = [FV_{t-1} (1 + k) (1 + i) – C_{t-1} – FV_t] / (1 + j)$$

Therefore $FV_{t-1} [(1 + k)(1 + i) – k (1 + j)] = C_{t-1} + FV_t$

So $\quad FV_{t-1} [(1 + i) – k (j - i)] = C_{t-1} + C_t / [1 + i – k (j – i)]$

So $\quad FV_{t-1} = (C_{t-1} + C_t / [1 + i – k (j – i)]) / [1 + i – k (j – i)]$

The nature of the recursive formula here is obvious, and we may write

$FV = DBE_{i*}$, the discounted best estimate at an interest rate $i^* = i – k (j – i)$.

It will be seen that the higher the capital loading required the higher the fair value will be, since the interest rate used to discount will be lower. Also, the higher the risk margin demanded on investment of capital the higher the fair value will be, as the interest rate used to discount will be lower. Both these results are obvious from the nature of the quantities, and it would have been a matter of concern to have had them contradicted by the result.

It may be noted that certain combinations of k and j-i will give values of i^* that are less than or equal to -100%, which will prevent the formula being used. For example, i = 4%, j = 14% and k = 10.4 will give i* = -100% exactly. This is not an invalid result: what it demonstrates is that with the level of capital commitment required, there is no possible level of fair value that will return the purchaser the appropriate risk-adjusted rate of return.

4. How Much is the Fair-Value Liability?

The formula is simple. The fair value is equal to the discounted value of the liabilities, using a discount rate equal to i – k (j – i). The fair value will be higher than the discounted best estimate in all sensible formulations: for it to be otherwise there would need to be either a negative capital requirement or a risk-adjusted return on equity that was lower than the risk-free rate. It turns out that fair value is simply the best estimate of liabilities, discounted at a conservative rate of interest.

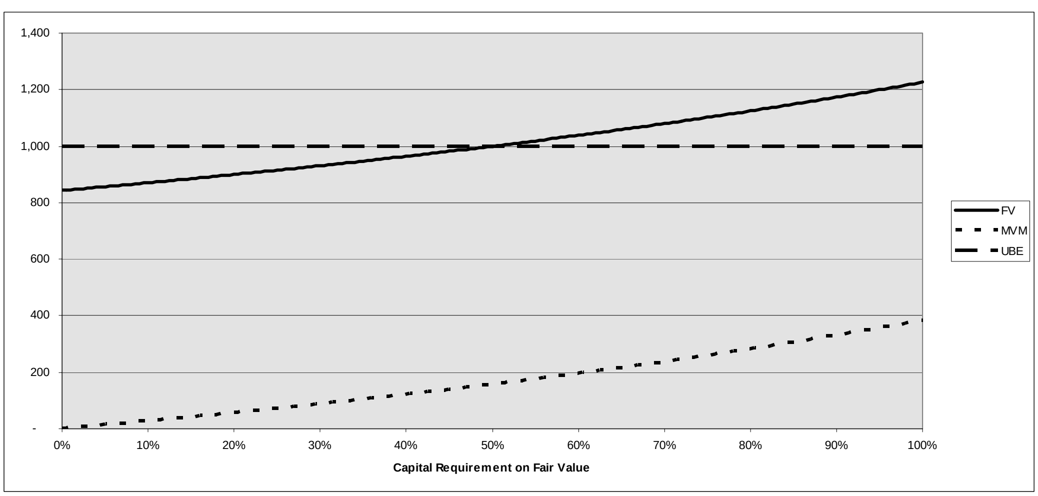
In most cases the fair value on this formulation will be less than the undiscounted best estimate reserve. For the reverse to be true, we need either a very low risk-free rate, or high capital requirements and a high adjustment for risk. The capital requirements suggested for reserves in CP190 ranged up to 17% of undiscounted reserves. A value of 25% for illustrative purposes would seem to be high. Similarly a 15% difference between a required rate of return and a risk-free rate would seem more than ample in most cases. This gives a positive adjusted rate that is greater than nil (and therefore a fair value that is less than the undiscounted best estimate reserve) at all risk-free rates of return greater than 3.75%.[4]

It is worth reiterating that although the allowance for risk is not explicit in this formulation, in the sense that there is not an explicit part of the formula that is the market-value margin, it is incorporated into the formula, and fully allows for the differential riskiness of various tranches of business. A highly risky set of liabilities will command values of k and j that are higher than a low-risk tranche. A set of liabilities that was certain (an unusual concept in general insurance) should justify using k=0 and j=i, and the fair value would be equal to the value of the liabilities discounted at the risk-free rate. Highly uncertain liabilities, on the other hand, would justify high values of j and k, and could lead to the use of a rate of interest that was less than nil to calculate the fair value. The important thing to bear in mind in considering the formula is that j and k are not constants over all tranches of liability, but could be thought of as functions of the riskiness of the business being valued.

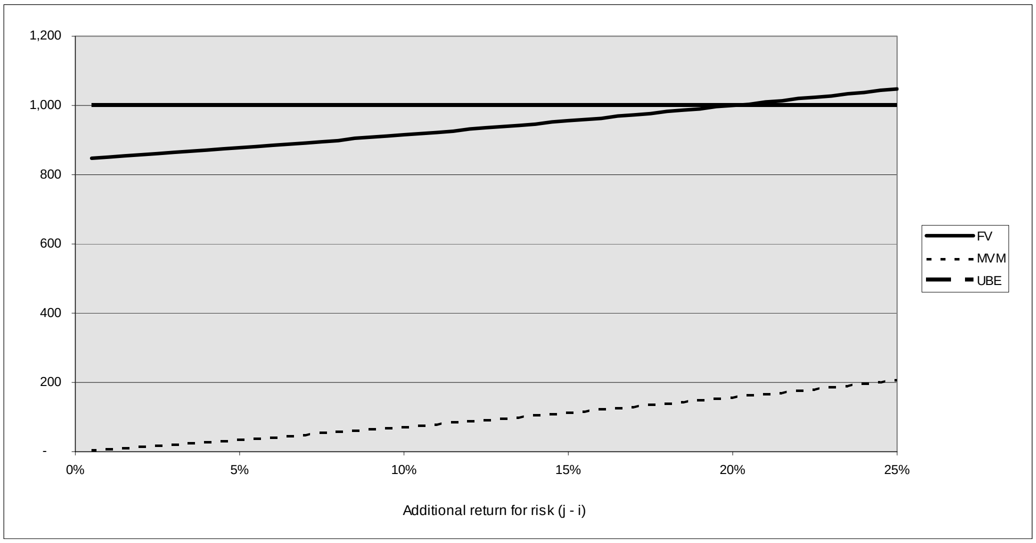
[4] The yields on government bonds currently available in, say, Japan and Switzerland, indicate that this would not be universally true in all economies at the present time.

Also, the fair value of a tranche of claims will not include any allowance for the risk that is diversified within the tranche being valued; if calculating the fair value of a single policy's claims then it is likely that a higher j and k will be required than when valuing a tranche of claims, and the fair value of claims arising from a book of business will be lower than the sum of the fair values of its individual components. This will apply until there is no risk diversification between the different books of business being valued. For a fuller discussion of this aspect see Clark et al, paragraph 7.2.4.[vi]

The graph below shows the fair value of liabilities and the market-value margin, and compares them to the undiscounted best estimate, for a portfolio of motor liability business. The portfolio is a real one, came from a European country, is fairly long-tailed, and has been scaled so that the total future claims payment is expected to be 1,000. For simplicity we have assumed that all claim payments are made at the end of the year. A risk-free interest rate of 4% has been assumed and a risk-adjusted rate of 12%. The graph shows the fair value as the capital requirement is increased. When there is no capital requirement the fair value will be equal to the best estimate of liabilities discounted at the risk-free rate; it rises gradually with the capital requirement, but not until the capital requirement reaches half the fair value does the fair value exceed the undiscounted best estimate of reserves. The market-value margin line is parallel to the fair-value line, being equal to the fair-value line less the best estimate discounted at the risk-free rate, which does not vary with the capital requirement.



The second graph shows the fair value for the same liability, with the capital requirement held constant at 20% of the fair value and the risk-free rate of return held constant at 4%, gradually increasing the required additional risk-adjusted rate of return. The fair value increases with the extra return allowed for risk, but does not exceed the undiscounted best estimate until around 20% extra is required.



The graph below shows the combinations of risk-adjusted rate of return and capital commitment that give a fair value for this set of liabilities that is equal to the undiscounted best estimate of liabilities. It should be noted that the values on the graph do not depend on the particular run-off pattern of the liabilities: equality will occur when the adjusted discount rate, $i - k(j - i)$, is nil, which occurs if $k = i / (j - i)$ or $j = i(1 + k) / k$. The "blob" in the middle of the bottom of the graph is intended to show the sorts of combinations of these two variables that might be considered to be typical and acceptable.[5] It will be noted that it lies wholly below and to the left of the line on the graph, indicating that, in general, we may expect fair values to be less than the undiscounted best estimate.

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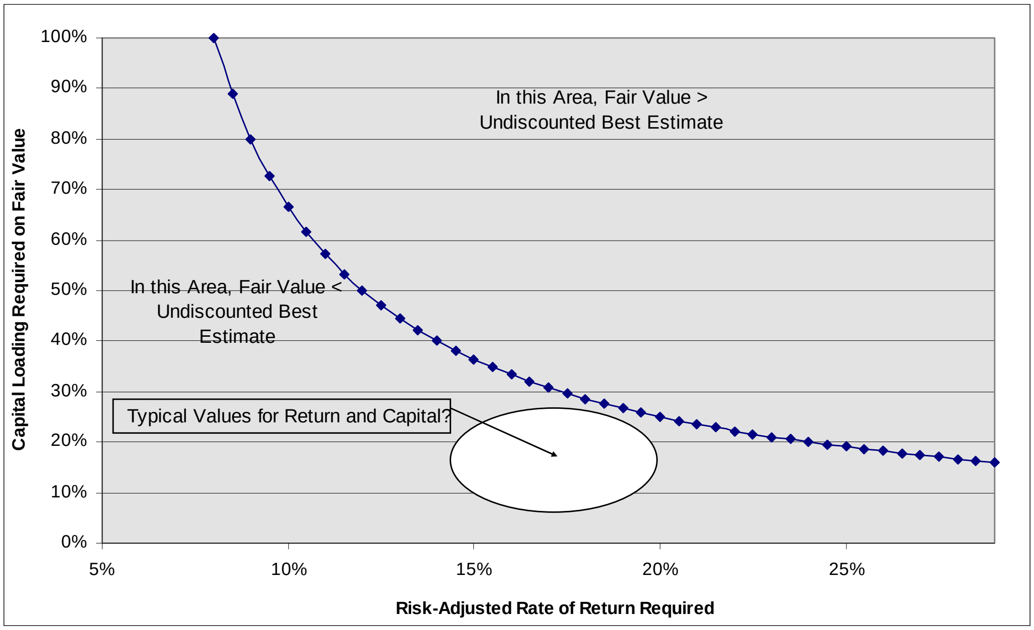
[5] The capital loading on the graph is a ratio of capital to reserves, and would be the capital required solely to run off liabilities for past claims. It should not be confused with the ratio of capital to premium commonly used as a yardstick for the capital adequacy of an insurance company writing new business. It may be noted that the capital loadings set out as the FSA's preferred factors in CP190 vary between 7.5% and 17%.



When fair values were first proposed, concern was expressed that the market-value margin would prove to be a major constraint on the conduct of business and that financing it would be a significant strain on companies. These formulae suggest that, for most business most of the time, there is unlikely to be any financing strain in the way expected, and that the market-value margin can be financed from the discounting of liabilities. These results do not depend on the particular set of liabilities chosen to illustrate them: they depend on the characteristics of the formula i – k (j – i), although the particular shapes of the first two graphs above will depend on the characteristics of the claims liabilities being modelled. This does assume, of course, that in setting undiscounted liabilities there has really been no implicit discounting, and that management would be happy to adopt the discounted value of their undiscounted reserves as the discounted best estimate.

It is worth restating at this point that this is a particular formulation of fair values, and that others might produce different results. However, the essence of the fair value of liabilities is that it is the value at which a willing buyer and a willing seller would be prepared to trade. Since the fair value as formulated provides an appropriately risk-adjusted rate of return to the purchaser of the liability and reflects the capital he will be required to subscribe, it is difficult to see how a much higher value than that required to give this risk-adjusted rate of return could be justified. Some very risky liabilities, asbestos-related runoff, for example, might justify a higher fair value, but if this cannot be justified on the basis that their degree of risk is so high that a very high capital loading, perhaps several hundred percent of the fair value, is needed, or else on the basis that their degree of risk is so high that a very high risk-adjusted return, perhaps several tens of percent, is justified, then any fair value that is higher than the formula appears to give the purchaser a return on his investment that is not justified.

5. How Does Profit Emerge?

If reserves are fully discounted, then profit emerges as premium is earned. At the end of the policy period reserves have to be set up to pay the discounted value of future claim payments; in subsequent years the release of reserve and the investment income earned on the reserve (assuming that the rate of interest used to discount the reserves is exactly achieved on the assets) will be exactly enough to make the payment of claims in the year and there is no profit or loss. Any profit or loss generated by the policy will have been recognised in full when the premium was earned.

If reserves are entirely undiscounted then the insurance profit emerges only at the end of the policy term. At that stage, a reserve must be set up that is sufficient to make all claims payments. (The reserve required up to that point may be based on premiums or claims; this is part of the entry value/exit value question and is outside the scope of this paper.) In subsequent years the release of reserves is sufficient to pay the claims, and any investment income on the reserve falls into profit. Therefore, compared with the discounted best estimate reserving strategy, a much lower profit (possibly a loss, even if the business is profitable over the whole period until the final liability is extinguished) is released over the period of the policy term followed by a string of profits as the reserve runs off.

If the most likely amount of the fair value liability is between the discounted and undiscounted best estimates then the profit emergence will be between these two extremes. This is illustrated by the following graph. It uses the same data as we have used to illustrate the fair value calculations, except that it relates to one year's business and not the liabilities produced by many past years. We have also the following assumptions.

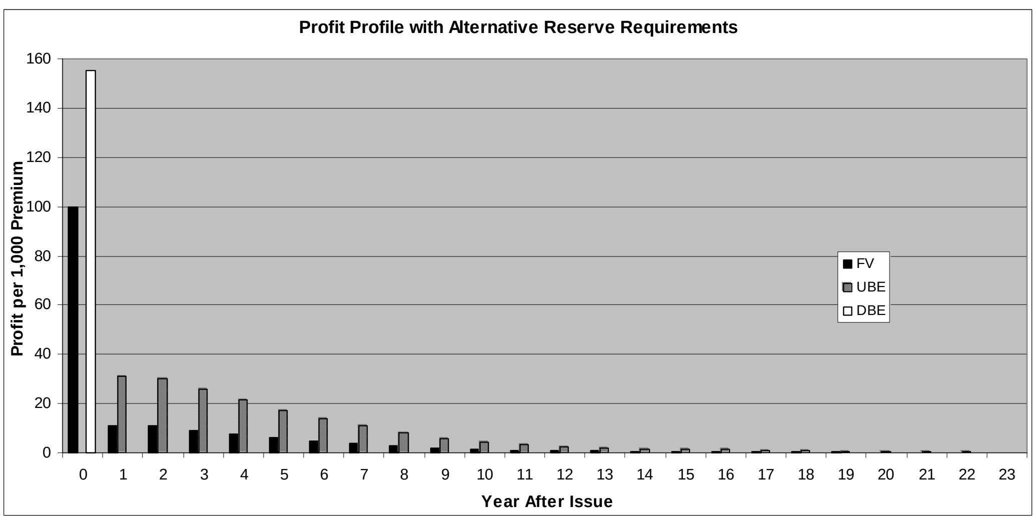
- Total premium = 1,000
- All policies written on 1 January.
- No claims are paid in the policy year itself, and all claims payments in subsequent years are made on 31 December.
- Risk-free rate of return : 4%
- Risk-adjusted rate of return for use in calculating fair value: 12%
- Capital required to be held on runoff of liabilities: 20% of fair value.

- Policy costs are 250, all payable when the policy is written. (Claims management costs are included in the claims amounts.)
- Claims are 780, payable from the second development year onward.

These assumptions are somewhat artificial, and have been selected to avoid the complications of dealing with policies that are partially expired. We shall examine only the accounting at each 31 December, which removes the need to estimate reserves for policies that are on risk at the balance date.

As the policies are written, the insurer receives a net (1,000 – 250) = 750; with interest at the risk-free rate the total assets increase to 780 at the end of the year. Since future claims are 780, setting up a claims reserve on an undiscounted best-estimate basis gives rise to nil profit or loss at that time; setting up a claims reserve on a discounted best-estimate basis will give risk to a profit, so long as the rate of interest used to discount exceeds nil.

The following graph shows the emergence of profit under the three assumptions.



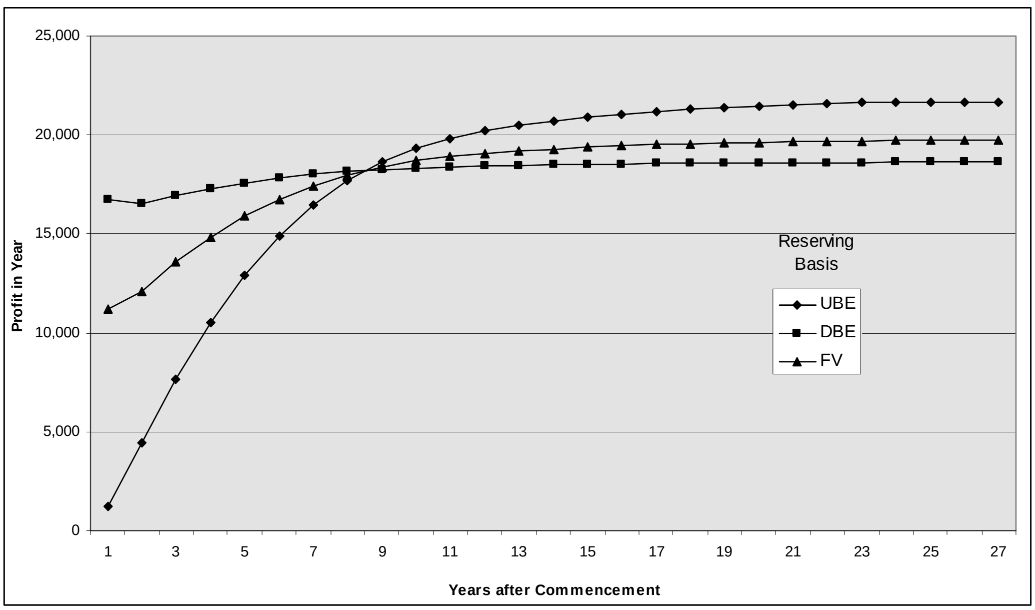
It is also worth pointing out that this business, using discounted best estimate reserves and with initial capital of 35% of premiums and capital during runoff of 20% of fair value of liabilities, gives the insurer a 26% return on its capital, despite having a combined ratio in excess of 100%. This is reduced to about 17% if profit emergence is delayed by the use of undiscounted best-estimate reserves. (The values of the two profit streams when discounted at the risk-free rate are equal, but delaying the release of profit and earning only the risk-free rate during the delay reduces the internal rate of return – in effect the extra capital tied up in the undiscounted reserve can earn profit only at the risk-free rate.)

Fair value liabilities are not intended to be used at the level of single policies, but at the level of tranches of business. We shall now consider the profit profile of a book of this business written over a number of years. The same characteristics have been assumed as for the single policy illustrated above, with total premium of 100,000 each year, and initial capital for a start-up business has been taken as 31,176. (This value was chosen so that no capital strengthening would be required to maintain the solvency criterion when using undiscounted best estimate reserves.) A solvency criterion was used that the company should maintain capital equal to 150% of the enhanced capital requirement as set out in CP190, calculated as 10% of the prior year's written premium plus 9% of claims reserves on an undiscounted best estimate basis at the end of the year, this being motor business. Any net assets that were surplus to this requirement at the end of the year were released as dividend. Fair values are calculated using a risk-adjusted rate of return of 12% and a capital loading of 20% of the fair value.

Since this company enjoys the happy position of writing a set amount of profitable business each year, it reaches a stationary position once the first year's claims have been paid in full. This being a long-tailed class of business, full payment does not happen until 26 years have elapsed. However, since the profit arises from investment income, using undiscounted reserves delays the emergence of profit, so that very little profit is earned in early years. On the other hand, if the reserves held are discounted at the risk-free rate of interest, which is what is assumed to be earned on the assets, the amount of profit released varies little from early years. (The reason it varies is that capital must be built up from retained profits so that the solvency criterion can continue to be met as the business expands, and this capital generates investment income. This investment income is profit, which gradually increases.) Because the example company has been assumed to operate under a single criterion for retaining capital that does not depend on the reserving basis, the amount of capital held is the same in each formulation, with the total assets being held being higher if undiscounted reserves are used.

Because the fair value of liabilities is simply the discounted value using a rate of interest that is intermediate between nil and the risk-free rate, the profit position is intermediate between using the discounted and undiscounted reserves. This result actually tells us more about the effects of discounting than it does about the effects of fair values.

The amounts of profit year by year are shown in the following graph. The slight discontinuities at the start for the discounted and fair-value positions are due to starting the model with the same amount of capital in each case: with discounted liabilities and the same capital criterion, this leaves excess capital at the end of the first year, which is released as dividend. As a consequence, the assets held in later years are lower, as is the investment income generated.



One serious reservation about this formulation is that the required level of capital should not be invariant whether liabilities are discounted or not. When the liabilities are undiscounted there is a substantial margin compared with the position with discounted liabilities. In effect, although the declared capital positions are the same in all three formulations, with the undiscounted position there is substantial hidden capital, at least theoretically and relative to the other positions, within the reserves. More assets are held that are available to meet the same liabilities. This is why the long-term profit level is higher with undiscounted liabilities than discounted liabilities: more assets generate more investment income. Also, the return on capital appears to be the same, since the same nominal level of capital is held; in fact the return on capital is lower when using undiscounted reserves because hidden capital is held as part of the reserves.

The fair-value approach does start from the premise that the real value of the liabilities is the discounted best estimate of the amount to be paid, although a rational purchaser would require a market-value margin in addition to this if he is to assume the liabilities. The question of whether reserves should be discounted or undiscounted is one that has been discussed widely by actuaries. This graph illustrates the difference to profitability that can arise from adopting one approach rather than the other: when the company reaches a steady state there is little difference but during the period that the reserves are being built up there can be substantial deferment of profit, the degree of deferment depending on the length of tail of the liabilities.

One of the reasons that actuaries generally feel uncomfortable with the discounting of liabilities is that profits are released quickly, before the doubt over the actual level of claims is resolved. This shows clearly in the graph, where there is substantial deferment of profit when reserving is on an undiscounted basis. The fair-value approach takes an intermediate path. It should be noted that if there is great doubt about the true level of claims then k and j will be higher than if there is little doubt, and the fair value will be closer to the undiscounted value than would be appropriate if the amounts of liability were considered to be close to certain. This seems to be a very sensible outcome. It may be contended that the fair-value approach addresses this particular actuarial objection to discounting claims reserves, with the level of discounting allowed being related to the level of risk.

In a particularly risky case $i - k(j - i)$ will be negative, a reserve will be required that is greater than the undiscounted best estimate and the emergence of profit will be delayed even beyond the undiscounted reserves case. In an extreme case, it could be less than -100%, and there will be no fair value.

Another point to note is that as k is increased, not only does the fair value increase so that there is an extra margin in the reserves, but the capital that must be held alongside it will also increase, so that there will be two sources of the increase in total resources.

6. What About the Volatility?

The analysis above has been carried out on a company operating in a fixed environment where the parameters never change. Much of the criticism of the move to fair values is that it is likely to cause great volatility in insurance companies' reported results. It is worth examining this in the context of our example company, which writes fairly long-tailed business, always achieving the same loss ratio, and earning a risk-free rate of return on its assets.

Within this context, the main item that will vary from year to year is the risk-free rate of return. Even without varying the basis for the valuation of reserves, this implies a significant degree of volatility in the results of the company: the company's profit is derived entirely from investment income, and a change in the rate earned on assets affects the profit directly. In effect, we have assumed that the company holds all its assets in cash, which is a significant mismatch between assets and liabilities. The effect of varying this assumption will be investigated later.

In the examples above, we have used 4% as the risk-free rate of return. This was chosen purely for illustrative purposes. In order to illustrate the effects of volatility we have generated scenarios for economic modelling purposes and taken the yield on invested cash in the United Kingdom as the risk-free rate of interest.[6] One thousand scenarios were generated, and the average yield in the first year was 3.9645%, which fits well with a starting value of 4%. We have kept the capital loading, k, and the extra return to allow for risk, j-i, constant.

The graph below shows the distribution of profit in year 27, a couple of years after a steady state has been reached. The actual graphs have been somewhat smoothed.

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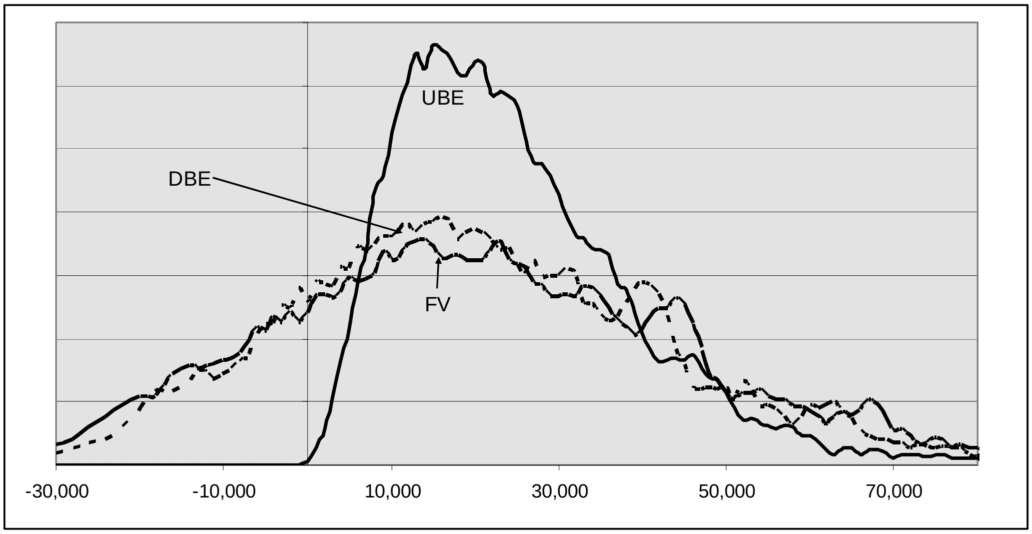
[6] The scenarios were generated on Global Cap:Link, a proprietary Towers Perrin economic scenario generator. Technical and other details are available on request.



The undiscounted best estimate basis shows the least variation in reported profit. This is not at all surprising: the only source of variation is the actual interest earned on assets. It may be thought to be more surprising that the profits on a fair-value basis are actually slightly more erratic than those on a fully discounted basis. The tails of the distribution are slightly fatter, and the standard deviation somewhat higher. It might be thought to be surprising because, up to this point, we have come to view the fair value as a compromise between discounted and undiscounted reserves, at least for business that is not so risky that it justifies a negative rate of interest to assess fair values. However, that is true only at a single point in time. If we consider the moving interest rates, we have used a constant extra return for risk and a constant capital loading on the fair value, which means that the adjustment to the risk-free rate, –k (j – i), is constant, so that the risk-free rate and the adjusted rate used to calculate fair value move in parallel with each other.[7] However, there is no variation at all in the

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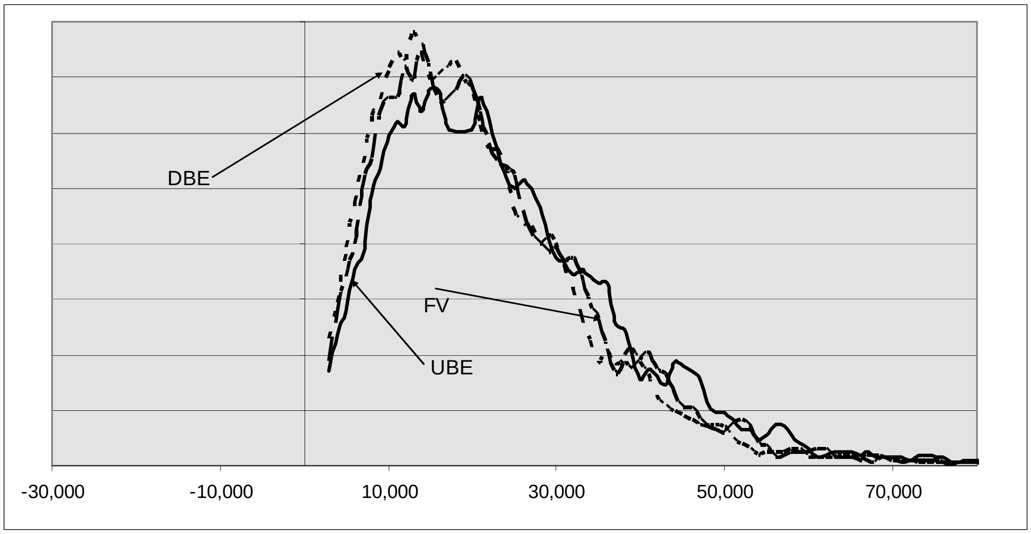
[7] The assumption that the adjustment to the risk-free rate of interest should be constant is not unchallengeable. If the fair value of assets depends on the risk-free rate of interest it is quite possible that k, the capital loading on the fair value, should also vary in some way with the risk-free rate. Similarly, to quote principle 3 from White et al, "A precise method is set down for determining the rate of return to be required over and above the risk-free rate. This extra return could take into account financial market conditions." This implies that, while it is j – i (in our notation) that is specified, it is not necessarily invariant to market conditions, one of which is the level of i. We have not investigated this possibility further.

rate used to value the undiscounted best estimate, since no rate enters the calculation. Also, since the adjusted interest rate is lower than the risk-free rate, the fair value is more sensitive to changes in the risk-free rate of interest than is the discounted best estimate.

The following two graphs show the variability of results, firstly, due to the variation in reserves because of changes in interest rates without varying the actual rate of interest earned on the assets and, secondly, due to varying the interest received without varying the basis on which liabilities are valued. The two sets of results have been generated in the same way as the overall results, except that in one case the interest rate received was held constant throughout at 4% and in the second the interest rates used to value the liabilities were held constant at 4% and 2.4% (= 4% - 20% × (12% - 4%). In the second case there is no variation in the results from the undiscounted best estimate basis: this valuation basis does not change with variations in the risk discount rates, and we are not varying the interest received from scenario to scenario.

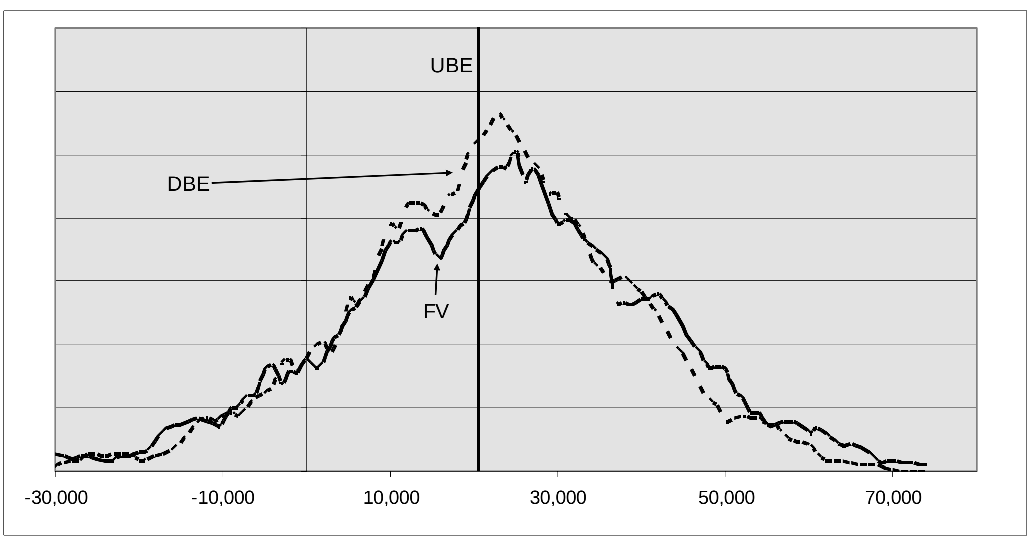
It should be noted that the two sets of results are not additive. There is interaction between the two effects that are being varied; the amount of profit generated, the amount of assets held and, therefore, the amount of investment income generated, depend on interactions between the two quantities being varied.

The following graph shows the variability in results when only the interest rate earned on assets is varied.



There is little difference between these distributions. The distribution of the profit with undiscounted best estimates is slightly more spread out, but this is a consequence of holding more assets under this dispensation. Since more assets are held, more investment income, and therefore more profit, is generated than in the other cases. The sample coefficients of variation were all between 0.643 and 0.656.

The following graph shows the variability in results when only the interest rate used to estimate reserves is varied.



It is the third of these three results, the one found by varying the valuation rate of interest alone, that is of most significance to us. Investment income is variable without the introduction of fair values of liabilities, but all the variability in the third of the graphs above is a result of introducing discounting, and fair value reserves are discounted reserves.

There is considerable variability between the results in this exercise. The use of fair values is enough to give rise to losses rather than profits in 130 (out of 1,000) cases for the discounted best estimate and 142 cases for the fair value, despite the fact that the company is writing business that produces an absolutely reliable profit of 18.6% of premiums when reserves are held on a discounted best-estimate basis (when the risk-free rate is 4%) and capital is held of 150% of ECR. So why should the adoption of this method of valuing liabilities cause losses to arise so often when the business is fundamentally so profitable? It is because the profit is only 4.8% of reserves, when a steady state has been reached, and a 1% variation in interest rates will cause a variation in reserves of between about 4% and 4½%. Variations of 1% in the rate of interest occur in 29% of years in the scenarios generated, and when these are reductions it can be enough to cause the company to register a loss.

This company has retained 150% of the required capital under the ECR, distributing any profit above this level as dividend. Any loss in a year will cause this target not to be met at the end of the year. However, a profit the following year will allow it to be restored, at least partially.

The ECR regime has not yet been implemented, indeed may not be implemented in the proposed manner, and it is far from clear now what the market will come to accept as reasonable levels of capital. Presumably companies will want to hold somewhat more than a generally-acceptable level; if a company really were targeting 150% it would be because it could slip below this from time to time without consequences that were too adverse. In the model, the company will not pay any dividend when it makes a loss, but because its prior year's dividend will have reduced its ECR cover to 150% it will automatically fall below this level in the year of making a loss.

In the thousand scenarios the numbers of times the cover for the ECR fell below certain levels in the year 27, with variation in valuation interest rates but not in the rate earned on invested funds, are shown in the following table.

Level of Cover for ECR	Number of Instances (out of 1,000) in which Cover Level Breached	
	Based on Fully Discounted Reserve	Based on Fair Value
150%	163	182
125%	65	82
100%	19	30
90%	7	18
75%	3	8
50%	1	1

In the one instance in which capital fell below 50%, it actually fell to 25% and 3% for discounted and fair value respectively. This scenario was one in which interest rates had increased substantially and then fell for several years in a row. As a result, profits were made as interest rates increased, they were distributed so that the capital held met the 150% criterion, and there was no buffer to absorb the impact of a return to lower interest rates.

It may be concluded that the adoption of fair values is not likely to bankrupt a company, but might well come close.

This may be considered unrealistic: what really brings the company close to bankruptcy in this example is a large over-distribution of dividend in the ten years before interest rates started to fall. This reduced the value of the liabilities, but implicit in the reduced value was a requirement to earn interest at a commensurate rate to fund the unwinding of the reserves. When interest rates fell, this became impossible. It would be unlikely in practice that a company would stick to such a rigid dividend policy, being fully aware that the source of its excess profits could not be relied upon. Distributing only part of its excess earnings would have built up a larger buffer on the ECR in years when the interest rate was rising. Different, and generally more conservative, dividend policies would have the effect of building up such a buffer. On the other hand, in the interests of simplicity taxation has been ignored in this model. In the real world the company would have had to distribute some of its profits as tax while interest rates were rising, although it is also possible that it might have been able to reclaim some of the tax paid when it subsequently began to make losses as interest rates fell.

It is clear that adopting fair value has the capacity to cause substantial variability in recorded profitability. Whether this appropriately reflects reality or demonstrates the inappropriateness of fair value accounting is a philosophical question, not a numerical one. It is true that the model used to demonstrate this has been of a company whose assets and liabilities are completely mismatched; this structure was adopted to allow us to investigate the variability of fair-value claims reserves.

7. The Asset Side

In the modelling so far we have assumed that the company receives interest on its assets at the risk-free rate of interest. This leads to volatility in investment earnings. The fair valuation of liabilities also leads to substantial volatility in recorded profit. However, fair valuation is meant to be applied to both sides of the balance sheet.

If the company really held all its assets as cash, as we have assumed in the modelling so far, then the fair value is simply the amount of money held on deposit. However, this is not an optimal asset-management strategy, at least if the company wishes to minimise volatility. (If it had a strong view that interest rates were about to rise and was prepared to invest to maximise profit on its view then it would be a different matter. It would wait until after the interest rate rise and then invest.)

The opposite extreme investment policy, which we should probably regard as the natural position, would be to invest the assets backing the reserves in gilts whose proceeds exactly matched the liabilities.[8] It should be noted that the value of the liabilities would be greater than the value of the assets, as the liabilities are valued at less than the risk-free rate of interest in order to provide an appropriate market-value margin; the assets are valued at the risk-free rate of interest, and if corporate bonds were used they would yield a higher rate of interest and have a lower market (and, therefore, fair) value. No asset that precisely matches the liabilities in this sense is available: to be discountable at an interest rate that was lower than risk-free it would have to be less risky than a risk-free asset, which is by definition impossible.

Since liabilities need to be valued at a lower rate of interest than assets (or, to be precise, are valued at a lower rate of interest than is implicit in the market value of the assets) their value will be more sensitive to movements in the risk-free rate of interest

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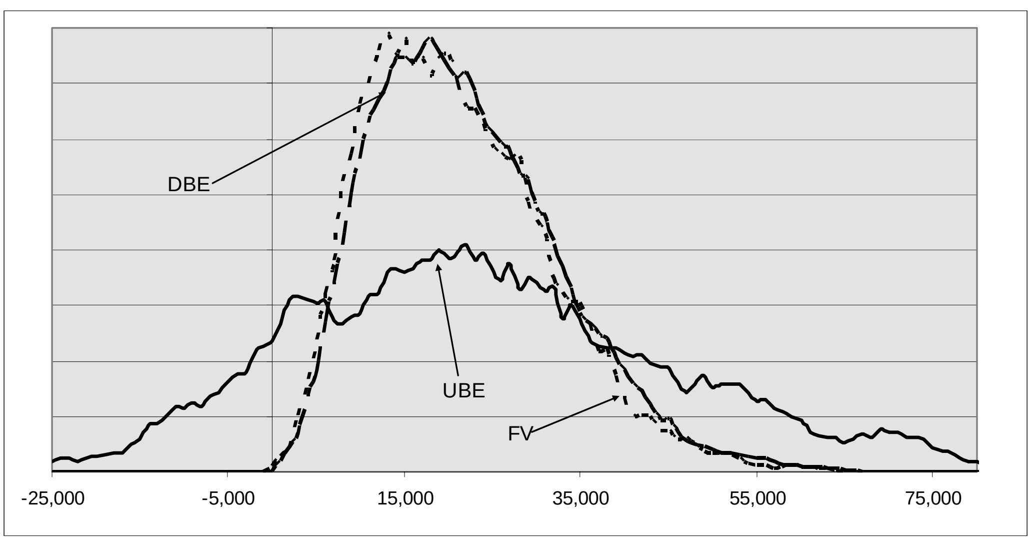
[8] Suggesting this strategy implies that such gilts exist and that they all yield the risk-free rate of interest or, equivalently, that the risk-free yield curve is flat, and the gilt yield is the same as the risk-free rate at all durations. In the model this has been adopted as an implicit assumption. What rate should properly be used for a risk-free rate is a matter of some discussion in financial economics; it is outside the scope of this paper.

than the value of assets. This means that, even if asset proceeds and liability outgo are perfectly matched, there will be a small profit when interest rates rise and a small loss when they fall. As the reserve runs off, if there is no change in the risk-free rate of interest there will be a profit released every year as a larger discount unwinds on the assets than the liabilities.

The following graph shows the distribution of profit in year 27 if the investment portfolio of the office follows the following strategy:

- each year assets are purchased that will exactly meet the claims payments in future years;
- these assets yield the risk-free rate of return at the time they are purchased (which is the start of the year, when the policy proceeds are received);
- any remaining assets are invested in cash, which each year yields the risk-free rate applicable at the end of the year;
- cash can become negative or, in other words, the company can borrow at the risk-free rate to fund its matched strategy.



It will be observed that this time it is the undiscounted-best-estimate basis that yields the most volatile result. In effect, this case is the converse of the case using discounted reserves and investing in cash; in that case the liability side of the balance sheet was sensitive to economic conditions and the asset side invariant while in the new case it is the other way around; either way the two are not matched. The other two have much less variability. This is scarcely surprising: having matched asset and liability outgo the discounted values of the two should move in parallel.

The result is not invariant to the risk-free rate: a high rate of interest generates more interest from the assets held as cash and reduces the cost of buying the assets that will fund the claims as they fall due. The business is genuinely more profitable when interest rates are high. In reality we might expect high interest rates to be associated with periods of higher inflation and therefore, perhaps, higher claims costs or, if the business were genuinely more profitable the premium rate is likely to be bid down.

8. Conclusion

The White et al principles are sensible and consistent with the market-consistent exit-fair-value concepts, at least if we assume that it is appropriate for margins to be available to reflect diversifiable risk. With reasonable assumptions about the release of capital committed to support liabilities they imply a simple formula to be used to calculate the fair value of liabilities. This is that the fair value of claims liabilities is equal to the present value of those liabilities, the interest rate to be used for this calculation being the risk-free rate of interest, minus the extra return to allow for risk multiplied by the capital loading required as a proportion of the fair value. This rate will always be no more than the risk-free rate. Under assumptions that are likely to be sensible for most tranches of insurance business it is likely to be more than nil, which means that the fair value of liabilities will be between the undiscounted best estimate and the best estimate discounted at a risk-free rate of interest. In exceptionally risky categories of business it may be more than the undiscounted best estimate. The consequences of using fair-value accounting will therefore be those that are associated with using discounted reserves for accounting.

The fair value of liabilities will vary with the risk-free rate of interest. For reasonably long-tailed liabilities this volatility will be substantial, and may be enough to wipe out the profit on a year's trading, even though the underlying business is comfortably profitable. In extreme cases it may be enough to threaten solvency. However, if the company has an investment portfolio in which assets are matched to liabilities, then the volatility of the results is much reduced, and reflects only those real changes to profitability that are a result of yields on held assets being high or low.

Appendix 1 Alternative Formulations

In this appendix, we consider the implications of reducing the total held assets in proportion to the discounted best estimate of claims and the undiscounted best estimate of claims.

Discounted Best Estimate At the end of the year, the total assets held are reduced in proportion as the claims discounted at a risk-free rate of return have reduced. Therefore, the assets held at the start and end of year t will be

$$\frac{(FV + K)}{\sum_{n=1}^{\infty} C_n v_i^n} \sum_{s=t}^{\infty} C_s v_i^{(s+1-t)} \quad \text{and} \quad \frac{(FV + K)}{\sum_{n=1}^{\infty} C_n v_i^n} \sum_{s=t+1}^{\infty} C_s v_i^{(s-t)}$$

where i is the risk-free rate of interest and $v_i = (1 + i)^{-1}$.

The opening fund generates interest at the risk-free rate but the claims in year t also have to be paid. Therefore the income to the purchaser in year t is given by

$$\frac{(FV + K)}{\sum_{n=1}^{\infty} C_n v_i^n} \sum_{s=t}^{\infty} C_s v_i^{(s+1-t)} (1 + i) - \frac{(FV + K)}{\sum_{n=1}^{\infty} C_n v_i^n} \sum_{s=t+1}^{\infty} C_s v_i^{(s-t)} - C_t$$

$$= \frac{(FV + K)}{DBE_1} \left(C_t + \sum_{s=t+1}^{\infty} C_s (v_i^{(s-t)} - v_i^{(s-t)})\right) - C_t$$

$$= \frac{(FV + K)}{DBE_1} C_t - C_t$$

$$= C_t \left((FV + K) / DBE_1 - 1\right), \text{ where :}$$

$$\sum_{t=1}^{\infty} C_t v_i^t = DBE_1 \text{ (best estimate discounted at risk-free rate)}$$

In order for the value equation to work, we require that K be equal to

$\sum_{t=1}^{\infty} C_t \left((FV + K) / DBE_1 - 1\right) v_j^t$, where j is the appropriate risk-adjusted rate of return and v_j is defined accordingly.

So $K = \left((FV + K) / DBE_1 - 1\right) \sum_{t=1}^{\infty} C_t v_j^t = \left((FV + K) / DBE_1 - 1\right) DBE_2$, where

$$DBE_2 = \sum_{t=1}^{\infty} C_t v_j^t \quad \text{(best estimate discounted at risk-adjusted rate)}$$

$= (FV + K - DBE_1)\, DBE_2 / DBE_1$

So $FV + K - DBE_1 = DBE_1 / DBE_2\, K$

So $FV = DBE_1 + K\,(DBE_1 - DBE_2) / DBE_2$ [Result 1]

From which we see that is the market-value margin $MVM = K\,(DBE_1 - DBE_2) / DBE_2$.

It will be seen that the market-value margin is proportional to K.

It will also be seen that the smaller DBE_2 is in relation to DBE_1, the larger is the fraction $(DBE_1 - DBE_2) / DBE_2$, and the higher the market value margin will be. Increasing j will reduce DBE_2 but will have no effect on DBE_1, increasing this fraction. These two results mean that the higher the premium return demanded for risk the higher the fair value will be, and the higher the capital commitment the higher the fair value. Both these implications are what we would have expected, and if the formula had not confirmed them it would have been very surprising, to the extent of throwing the formula itself into question.

We consider now the implications of setting the required initial level of capital as a proportion of the discounted value of claims. This means that $K = k\,FV$. Putting this into Result 1 we get $FV = DBE_1 + k\,FV\,(DBE_1 - DBE_2) / DBE_2$, which simplifies to

$$FV = DBE_1 / (1 - k(DBE_1 / DBE_2 - 1)) \qquad \text{[Result 2]}$$

Result 2 seems to be the simplest statement of the result, although it may also be stated as

$$DBE_1 \left(1 + \frac{k\,(DBE_1 / DBE_2 - 1)}{(1 - k\,(DBE_1 / DBE_2 - 1))}\right)$$

This second formulation is more complicated, but it does give an explicit formula for the market-value margin, if we omit the "1+".

It will be noted that this formula is valid only when $k < DBE_2 / (DBE_1 - DBE_2)$. If the required risk-adjusted rate of return or the capital commitment is sufficiently high then there is no fair value that will yield the required return.

Undiscounted Best Estimate In this approach we assume that the total assets will be reduced in proportion to the undiscounted best estimate. The algebra is similar to Approach 1.

The release to profit in year t (t=1,2, …) is equal to the reduction in total assets required to be held, plus interest at the risk-free rate, minus claims payments. This is equal to:

$$\frac{(FV + K)\sum_{s=t}^{\infty} C_s\,(1 + i)}{\sum_{n=1}^{\infty} C_n} - \frac{(FV + K)\sum_{s=t+1}^{\infty} C_s}{\sum_{n=1}^{\infty} C_n} - C_t$$

$$= \quad (FV + K)\left(C_t\,(1 + i) + i\sum_{s=t+1}^{\infty} C_s \right) / \sum_{n=1}^{\infty} C_n \quad - C_t$$

$$= \quad (FV + K) / UBE\left(C_t\,(1 + i) + i\sum_{s=t+1}^{\infty} C_s \right) - C_t$$

$$= \quad (FV + K) / UBE\left(C_t + i\sum_{s=t}^{\infty} C_s \right) - C_t$$

In order for the value equation to work, we require that K be equal to

$$\sum_{t=1}^{\infty} v_j^t \left((FV + K) / UBE \times \left(C_t + i \sum_{s=t}^{\infty} C_s \right) - C_t \right)$$

$$= \sum_{t=1}^{\infty} v_j^t (- C_t) + \sum_{t=1}^{\infty} v_j^t \ (FV + K) / UBE \times \left(C_t + i \sum_{s=t}^{\infty} C_s \right)$$

$$= - DBE_2 + (FV + K) / UBE \sum_{t=1}^{\infty} v_j^t \left(C_t + i \sum_{s=t}^{\infty} C_s \right)$$

$$= - DBE_2 + (FV + K) / UBE \left(DBE_2 + \ i \sum_{t=1}^{\infty} v_j^t \sum_{s=t}^{\infty} C_s \right)$$

$$= - DBE_2 + (FV + K) / UBE \left(DBE_2 + \ i \sum_{t=1}^{\infty} C_t \sum_{s=1}^{t} v_j^s\right)$$

$$= - DBE_2 + (FV + K) / UBE \left(DBE_2 + \ i \sum_{t=1}^{\infty} C_t \,(1 - v_j^t) / j\right)$$

$$= - DBE_2 + (FV + K) / UBE \left(DBE_2 + \ i/j \sum_{t=1}^{\infty} C_t \,(1 - v_j^t)\right)$$

$$= - DBE_2 + (FV + K) / UBE \left(DBE_2 + \ i/j \,(UBE - DBE_2)\right)$$

$$= (FV + K) / j \ UBE \left(i \ UBE + \ (j - i) \ DBE_2\right) - DBE_2$$

$$= (FV + K) \left(i + \ d \,(j - i)\right)/j - DBE_2$$

where $d = DBE_2 / UBE$

This gives $jK = (FV + K) \left(i + \ d \,(j - i)\right) - jDBE_2$ [Result A.] This result will be used below.

This may easily be solved to give

$$FV = \left((j - i) \,(1 - d) \,K + j \ DBE_2\right) \ / \ \left(i + (j - i) \,d\right)$$

For this to give a valid solution, $i + (j - i)\, d$ must be positive. As all the elements in the formula are positive, it would appear that this condition is always met.

Turning to a couple of special cases for K, where it is proportional to FV and proportional to UBE, we get the following.

Let K = kFV

Then, from Result A, $jk\ FV = (1 + k)\ FV\ \left(i + d\ (j - i)\right) - jDBE_2$

So $FV = jDBE_2\ /\ ((1 + k)(i + (j - i)\ d) - jk)$

Let K = kUBE

Then, from Result A, $jk\ UBE = (FV + k\ UBE)\ \left(i + d\ (j - i)\right) - jDBE_2$

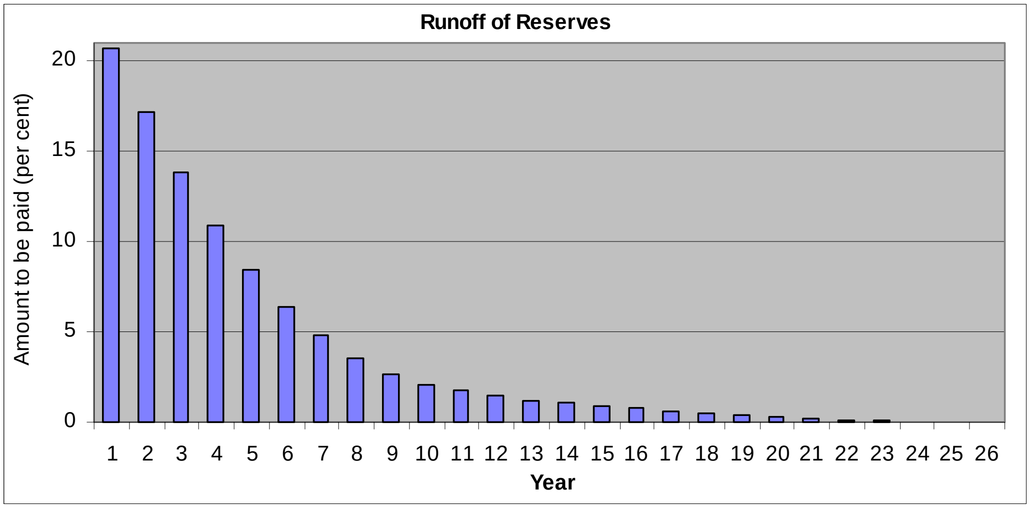
So $FV = (k\ (j - i)\ UBE + ((1 - k)\ j + ki)\ DBE_2)\ /\ (i + (j-i)\ d)$

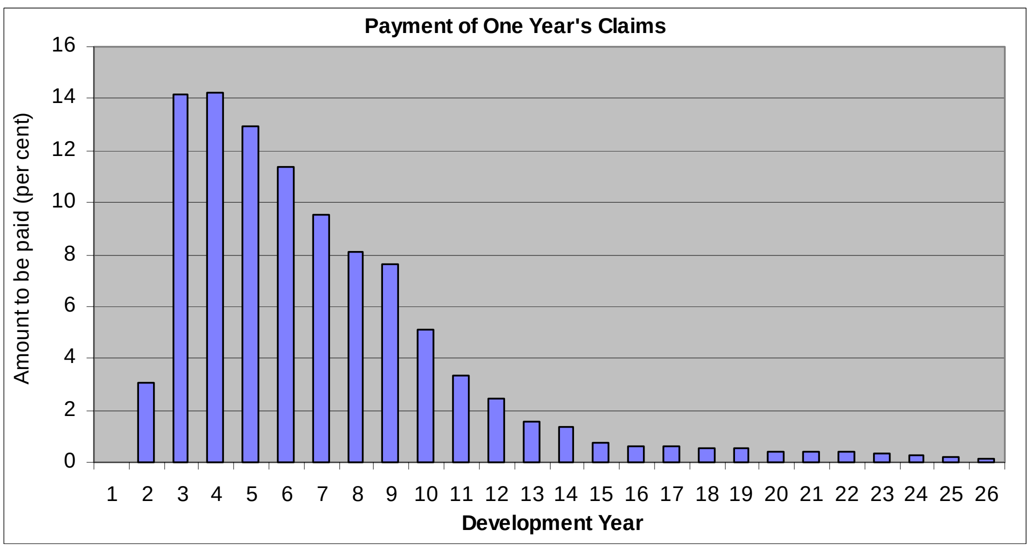
Appendix 2 Exhibits

Fair Values in General Insurance
Payment Patterns Used in Illustration

Development year	Runoff of Reserves	Runoff of One Year's Claims %	Development year	Runoff of Reserves	Runoff of One Year's Claims %
1	20.74	-	14	1.07	1.38
2	17.22	3.03	15	0.92	0.73
3	13.79	14.18	16	0.79	0.62
4	10.89	14.22	17	0.64	0.59
5	8.41	12.95	18	0.51	0.56
6	6.42	11.35	19	0.40	0.51
7	4.84	9.56	20	0.28	0.42
8	3.51	8.08	21	0.19	0.40
9	2.68	7.59	22	0.11	0.38
10	2.11	5.09	23	0.07	0.36
11	1.72	3.37	24	0.03	0.30
12	1.45	2.43	25	-	0.20
13	1.22	1.56	26	-	0.13

Payment Patterns Used in Illustrations





Fair Values in General Insurance

Illustration of Fair-Value Calculation

Risk free rate	4%	Fair value of liability	899,782
Risk-appropriate rate	12%	Capital commitment	179,956
Capital loading on fair value	20%	Market-value margin	56,435
Adjusted interest rate	2.400%		

Calendar Year	Claim Payments	Discount factor at 4%	Discounted Estimated Unpaid Claims	Opening Cash Balance	Claim Payments	Interest Received	Release of Surplus	Risk-adjusted discount factor	Discounted Surplus
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
2003	207,436	0.962	199,457	1,079,739	207,436	43,190	58,763	0.893	52,467
2004	172,156	0.925	159,168	856,729	172,156	34,269	48,139	0.797	38,376
2005	137,878	0.889	122,573	670,703	137,878	26,828	38,307	0.712	27,266
2006	108,855	0.855	93,050	521,346	108,855	20,854	30,113	0.636	19,137
2007	84,068	0.822	69,098	403,233	84,068	16,129	23,265	0.567	13,201
2008	64,231	0.790	50,763	312,029	64,231	12,481	17,839	0.507	9,038
2009	48,439	0.760	36,810	242,440	48,439	9,698	13,567	0.452	6,137
2010	35,117	0.731	25,659	190,132	35,117	7,605	10,065	0.404	4,065
2011	26,761	0.703	18,802	152,555	26,761	6,102	7,793	0.361	2,810
2012	21,086	0.676	14,245	124,103	21,086	4,964	6,203	0.322	1,997
2013	17,232	0.650	11,194	101,778	17,232	4,071	5,075	0.287	1,459
2014	14,462	0.625	9,033	83,542	14,462	3,342	4,229	0.257	1,085
2015	12,161	0.601	7,304	68,193	12,161	2,728	3,523	0.229	807
2016	10,713	0.577	6,186	55,236	10,713	2,209	3,026	0.205	619
2017	9,240	0.555	5,130	43,707	9,240	1,748	2,547	0.183	465
2018	7,859	0.534	4,196	33,668	7,859	1,347	2,110	0.163	344
2019	6,353	0.513	3,262	25,046	6,353	1,002	1,671	0.146	243
2020	5,141	0.494	2,538	18,023	5,141	721	1,317	0.130	171
2021	3,953	0.475	1,876	12,287	3,953	491	987	0.116	115
2022	2,807	0.456	1,281	7,838	2,807	314	687	0.104	71
2023	1,948	0.439	855	4,658	1,948	186	464	0.093	43
2024	1,132	0.422	478	2,432	1,132	97	265	0.083	22
2025	680	0.406	276	1,133	680	45	154	0.074	11
2026	293	0.390	114	344	293	14	64	0.066	4
2027	0	0.375	0	0	0	0	0	0.059	0
TOTAL	1,000,000		843,347	1,079,739	1,000,000	200,436	280,174		179,956

Fair Values in General Insurance

Model of Results: No change in conditions

Model begins 1 January 2004
Premiums are received and expenses paid in full every 1 January
Claims are all paid on 31 December

Investment income	4%	received every 31 December
Expense ratio	25%	
Loss ratio	78%	
Opening capital	31,176	
Required ECR cover	1.50	
Capital loading for FV	20.00%	
Risk discount rate	12.0%	

Undiscounted Best Estimate Basis

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Premiums	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
Expenses	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000
Claims	0	2,366	13,423	24,513	34,612	43,468	50,928	57,233	63,155	67,124	69,749	71,642	72,858	73,935
Reserves	78,000	75,634	64,577	53,487	43,388	34,532	27,072	20,767	14,845	10,876	8,251	6,358	5,142	4,065
Interest	4,247	7,417	10,619	13,507	15,935	17,905	19,473	20,702	21,645	22,319	22,812	23,187	23,476	23,709
Gross profit	1,247	4,417	7,619	10,507	12,935	14,905	16,473	17,702	18,645	19,319	19,812	20,187	20,476	20,709
Dividend	0	0	0	3,286	7,078	10,243	12,818	14,898	16,640	17,850	18,698	19,329	19,781	20,160
Profit	1,247	4,417	7,619	7,221	5,857	4,662	3,655	2,804	2,004	1,468	1,114	858	694	549
Capital	32,423	36,840	44,458	51,679	57,537	62,198	65,853	68,657	70,661	72,129	73,243	74,101	74,796	75,344
Reserves	78,000	153,634	218,211	271,697	315,086	349,618	376,690	397,458	412,303	423,179	431,430	437,789	442,930	446,995
Assets	110,423	190,473	262,669	323,376	372,622	411,816	442,544	466,114	482,964	495,308	504,673	511,890	517,726	522,340
Cash flow	79,247	80,051	72,196	63,993	56,324	49,437	43,545	38,469	33,490	30,195	28,063	26,545	25,618	24,774
ECR	17,020	23,827	29,639	34,453	38,358	41,466	43,902	45,771	47,107	48,086	48,829	49,401	49,864	50,230
Covered	1.9050	1.5461	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

Year	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
Premiums	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
Expenses	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000
Claims	74,506	74,988	75,452	75,888	76,288	76,618	76,933	77,231	77,509	77,746	77,899	78,000	78,000
Reserves	3,494	3,012	2,548	2,112	1,712	1,382	1,067	769	491	254	101	0	0
Interest	23,894	24,052	24,189	24,305	24,400	24,478	24,541	24,589	24,624	24,647	24,658	24,663	24,663
Gross profit	20,894	21,052	21,189	21,305	21,400	21,478	21,541	21,589	21,624	21,647	21,658	21,663	21,663
Dividend	20,422	20,646	20,845	21,019	21,169	21,292	21,397	21,486	21,558	21,612	21,644	21,663	21,663
Profit	472	407	344	285	231	187	144	104	66	34	14	0	0
Capital	75,816	76,223	76,567	76,852	77,083	77,269	77,413	77,517	77,583	77,618	77,631	77,631	77,631
Reserves	450,489	453,501	456,048	458,160	459,872	461,254	462,321	463,090	463,580	463,834	463,935	463,935	463,935
Assets	526,305	529,723	532,615	535,012	536,955	538,523	539,734	540,607	541,164	541,452	541,566	541,566	541,566
Cash flow	24,387	24,064	23,737	23,417	23,112	22,860	22,608	22,358	22,115	21,900	21,759	21,663	21,663
ECR	50,544	50,815	51,044	51,234	51,389	51,513	51,609	51,678	51,722	51,745	51,754	51,754	51,754
Covered	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

Fair Values in General Insurance

Model of Results: No change in conditions

Model begins 1 January 2004
Premiums are received and expenses paid in full every 1 January
Claims are all paid on 31 December

Investment income	4%	received every 31 December
Expense ratio	25%	
Loss ratio	78%	
Opening capital	31,176	
Required ECR cover	1.50	
Capital loading for FV	20.00%	
Risk discount rate	12.0%	

Discounted Best Estimate Reserving Basis

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Premiums	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
Expenses	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000
Claims	0	2,366	13,423	24,513	34,612	43,468	50,928	57,233	63,155	67,124	69,749	71,642	72,858	73,935
Reserves	62,489	62,622	54,071	45,143	36,851	29,468	23,187	17,810	12,600	9,135	6,875	5,257	4,251	3,344
Interest	4,247	6,521	9,434	11,946	14,040	15,748	17,114	18,187	19,012	19,596	20,020	20,340	20,584	20,782
Gross profit	16,758	16,532	16,940	17,289	17,578	17,812	17,999	18,145	18,257	18,337	18,396	18,441	18,475	18,503
Dividend	22,403	6,322	8,223	10,069	11,721	13,151	14,344	15,341	16,253	16,869	17,282	17,582	17,781	17,954
Profit	-5,646	10,211	8,718	7,221	5,857	4,662	3,655	2,804	2,004	1,468	1,114	858	694	549
Capital	25,530	35,741	44,458	51,679	57,537	62,198	65,853	68,657	70,661	72,129	73,243	74,101	74,796	75,344
Reserves	62,489	125,111	179,182	224,325	261,176	290,644	313,831	331,641	344,241	353,376	360,251	365,509	369,760	373,105
Assets	88,019	160,852	223,641	276,004	318,712	352,842	379,685	400,298	414,902	425,505	433,494	439,610	444,556	448,449
Cash flow	79,247	79,154	71,011	62,432	54,429	47,281	41,186	35,955	30,857	27,472	25,271	23,698	22,726	21,847
ECR	17,020	23,827	29,639	34,453	38,358	41,466	43,902	45,771	47,107	48,086	48,829	49,401	49,864	50,230
Covered	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

Year	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
Premiums	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
Expenses	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000
Claims	74,506	74,988	75,452	75,888	76,288	76,618	76,933	77,231	77,509	77,746	77,899	78,000	78,000
Reserves	2,907	2,541	2,179	1,830	1,504	1,234	968	709	459	240	97	0	0
Interest	20,938	21,073	21,191	21,292	21,377	21,446	21,503	21,547	21,580	21,601	21,612	21,616	21,616
Gross profit	18,525	18,544	18,560	18,574	18,585	18,594	18,602	18,607	18,612	18,614	18,616	18,616	18,616
Dividend	18,053	18,137	18,216	18,288	18,354	18,408	18,458	18,504	18,545	18,580	18,602	18,616	18,616
Profit	472	407	344	285	231	187	144	104	66	34	14	0	0
Capital	75,816	76,223	76,567	76,852	77,083	77,269	77,413	77,517	77,583	77,618	77,631	77,631	77,631
Reserves	376,012	378,553	380,732	382,562	384,066	385,299	386,267	386,976	387,435	387,675	387,772	387,772	387,772
Assets	451,828	454,775	457,298	459,414	461,148	462,569	463,680	464,493	465,018	465,293	465,403	465,403	465,403
Cash flow	21,432	21,085	20,739	20,404	20,088	19,828	19,569	19,316	19,071	18,854	18,712	18,616	18,616
ECR	50,544	50,815	51,044	51,234	51,389	51,513	51,609	51,678	51,722	51,745	51,754	51,754	51,754
Covered	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

Fair Values in General Insurance

Model of Results: No change in conditions

Model begins 1 January 2004
Premiums are received and expenses paid in full every 1 January
Claims are all paid on 31 December

Investment income	4%	received every 31 December
Expense ratio	25%	
Loss ratio	78%	
Opening capital	31,176	
Required ECR cover	1.50	
Capital loading for FV	20.00%	
Risk discount rate	12.0%	

Fair Value Reserving Basis

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Premiums	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
Expenses	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000
Claims	0	2,366	13,423	24,513	34,612	43,468	50,928	57,233	63,155	67,124	69,749	71,642	72,858	73,935
Reserves	68,035	67,301	57,860	48,158	39,216	31,300	24,592	18,877	13,408	9,761	7,370	5,654	4,573	3,606
Interest	4,247	6,743	9,843	12,506	14,721	16,524	17,963	19,093	19,960	20,576	21,025	21,365	21,625	21,836
Gross profit	11,212	12,075	13,560	14,835	15,894	16,756	17,443	17,983	18,397	18,692	18,907	19,069	19,194	19,295
Dividend	16,858	1,865	4,842	7,614	10,037	12,094	13,788	15,179	16,393	17,223	17,793	18,211	18,500	18,746
Profit	-5,646	10,211	8,718	7,221	5,857	4,662	3,655	2,804	2,004	1,468	1,114	858	694	549
Capital	25,530	35,741	44,458	51,679	57,537	62,198	65,853	68,657	70,661	72,129	73,243	74,101	74,796	75,344
Reserves	68,035	135,336	193,196	241,354	280,569	311,870	336,462	355,339	368,747	378,508	385,878	391,532	396,105	399,711
Assets	93,565	171,076	237,654	293,033	338,106	374,068	402,315	423,996	439,408	450,637	459,121	465,633	470,901	475,056
Cash flow	79,247	79,376	71,420	62,993	55,110	48,056	42,035	36,860	31,805	28,453	26,277	24,723	23,767	22,901
ECR	17,020	23,827	29,639	34,453	38,358	41,466	43,902	45,771	47,107	48,086	48,829	49,401	49,864	50,230
Covered	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

Year	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
Premiums	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
Expenses	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000
Claims	74,506	74,988	75,452	75,888	76,288	76,618	76,933	77,231	77,509	77,746	77,899	78,000	78,000
Reserves	3,121	2,714	2,315	1,935	1,582	1,290	1,005	732	471	246	98	0	0
Interest	22,002	22,146	22,271	22,377	22,466	22,538	22,598	22,644	22,677	22,698	22,710	22,714	22,714
Gross profit	19,374	19,443	19,503	19,554	19,596	19,631	19,659	19,681	19,696	19,707	19,712	19,714	19,714
Dividend	18,903	19,037	19,159	19,269	19,365	19,444	19,515	19,577	19,630	19,672	19,698	19,714	19,714
Profit	472	407	344	285	231	187	144	104	66	34	14	0	0
Capital	75,816	76,223	76,567	76,852	77,083	77,269	77,413	77,517	77,583	77,618	77,631	77,631	77,631
Reserves	402,833	405,547	407,862	409,798	411,380	412,669	413,675	414,406	414,878	415,123	415,221	415,221	415,221
Assets	478,649	481,769	484,429	486,649	488,462	489,938	491,088	491,923	492,461	492,741	492,853	492,853	492,853
Cash flow	22,496	22,158	21,818	21,489	21,178	20,920	20,664	20,412	20,168	19,952	19,810	19,714	19,714
ECR	50,544	50,815	51,044	51,234	51,389	51,513	51,609	51,678	51,722	51,745	51,754	51,754	51,754
Covered	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5

Appendix 3 References

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[i] Exposure Draft 5, Insurance Contracts. International Accounting Standards Board 2003. Available at http://www.iasc.org.uk/docs/ed05/ed05.pdf.

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[vi] The Implication of Fair Value Accounting for General Insurance Companies, P.K. Clark, P.H. Hinton, E.J. Nicholson, L. Storey, G.G. Wells and M.G. White, available at http://www.actuaries.org.uk/files/pdf/sessional/sm030324.pdf.